Paper III: Emergence and Structure
Length–Mass Reduction (LMR)

Jacob Rollins

Abstract

This paper develops the first populated structural layer of LMR: persistent configurations, their minimal inventories, and the emergence of composite persistence. The exposition remains pre-dynamical: no forces, fields, equations of motion, or energetic claims are assumed. Structure is presented as admissible routing, closure, reflection, phase retention, and torsion locking.

Contents

1 Introduction

LMR distinguishes grammatical primitives (Papers I–II) from realized structural populations (this paper). Paper III claims the territory of emergence: how minimal persistent configurations are enumerated and how composite persistence arises. Throughout, HF denotes half-fold count. "Resonance" is treated structurally as reflection and multiplicative composition of admissibility, not temporal oscillation. The grammatical primitives, corridor logic, and dual-length formalism used here are fixed in Papers I and II and are not re-derived in this work [**Rollins2025Codex**, **Rollins2026Persistence**].

Structural Discipline

Paper III proceeds strictly within the predynamical structural program fixed by Papers I and II. No forces, energetic functionals, or dynamical evolution laws are introduced. All π-multiplicities arise from admissibility engagement and retained structure, not from new routing operators. Throughout, structural arguments are predynamical.

Bridge to Paper I

Paper I established the codex diagrams as value-agnostic grammatical forms. The A-side and B-side are not physical domains or regions; they distinguish complementary representations within the dimensional grammar, and their correspondence is formalized by mirror inversion at mid_1. That codex-level correspondence remains unchanged in the present paper.

What changes here is not the grammar itself, but its first populated structural use. In the present paper, population proceeds primarily through the λ- and M'-faces of the fixed correspondence, with vertical movement carrying reciprocal inversion and horizontal movement carrying lm-mediated correspondence. These relations are grammatical before they are interpretive.

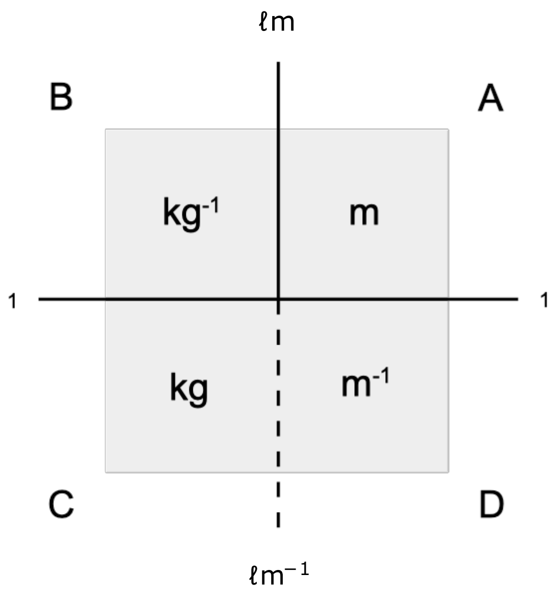

Figure 1: Codex quadrant grammar populated under the structural faces relevant to the present paper. Here m^{-1} is the inverse-length face later written as M', and kg^{-1} is the corresponding double-prime inverse-mass face. Vertical movement denotes reciprocal inversion; horizontal movement denotes lm-mediated correspondence. The diagram is grammatical and representational, not dynamical.

In the populated structural use adopted here, the meter-faced entries are read as λ and $M' = 1/\lambda$, while the kilogram-faced entries remain part of the fixed codex correspondence but are not the active working language of the present paper. Accordingly, the present paper works primarily in the λ- and M'-faces of the fixed codex correspondence, while leaving kg-faced realization and explicit double-prime deployment subordinate to later representational use.

The λ-face records admissible routing, half-fold inventory, and perturbative lattice structure. The M'-face records persistent configurations admitted under admissibility constraints. Mirror inversion at mid_1 therefore relates structural faces, not physical domains, and does not imply transport between meter- and kilogram-faced representations.

In particular, the same structural closure may appear as $(\sqrt{c})^4$ on the λ-face and as 4π curvature retention on the M'-face. These are not competing generators. They are mirror records of the same structural closure under the fixed codex grammar.

Bridge to Paper II

Paper II defined persistence as structural non-redistribution under admissible routing. A configuration is persistent when its half-fold inventory cannot be fully redistributed without violating admissibility.

Within that framework:

- the electron-class configuration realizes the minimal asymmetric persistent configuration (HF = 3),
- the proton-class configuration realizes the minimal reflective basin closure (HF = 4),
- the neutron-class configuration realizes torsion-retaining pseudo-closure under support.

Paper III does not revise the definition of persistence. It classifies the minimal persistent configurations admitted by the lattice under that definition.

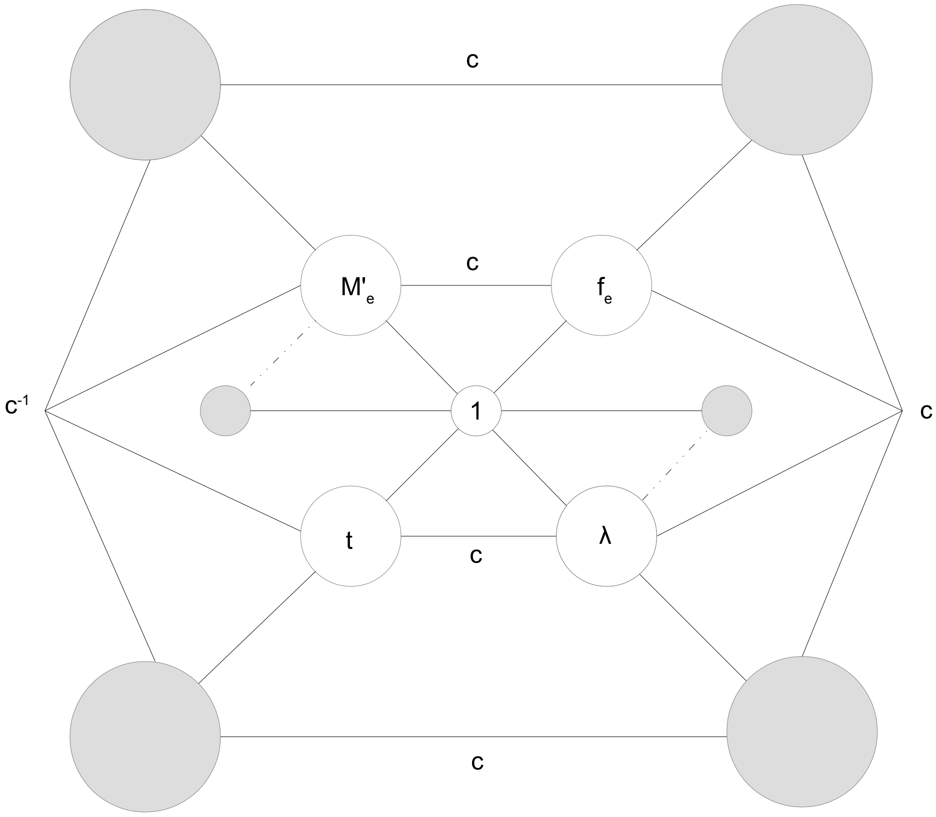

Figure 2: Extended Hourglass Diagram(greyed). Provided for orientation only. Not all relations shown are derived explicitly in this paper. Undefined corridors are intentionally muted and will be introduced in subsequent work.

Orientation Diagram (Non-Generative)

For orientation only, a greyed version of the complete LMR structural diagram is shown in Figure 2. This diagram is not generative. It introduces no primitives, relations, operators, or assumptions.

At this stage, many elements of the diagram are intentionally undefined or visually muted. The diagram should be read only as a topological placeholder indicating how the structures developed in this paper will ultimately cohere.

No argument in this paper depends on interpretation of the diagram. All relations depicted are earned explicitly in later sections and papers.

Remark 1.1 (On Diagrammatic Representations). All figures in this paper are schematic and representational. They encode admissible structural relations, half-fold inventories, and routing constraints, but do not imply spatial embedding, metric geometry, or physical visualization. Orientation, distance, and curvature shown in diagrams are not to scale and carry no independent physical meaning.

No normalization operator or global scaling functional is introduced at this stage. All references to global constraint are purely structural.

Figures are representational only: they encode admissibility and routing conditions and do not depict spatial geometry, intrinsic structure, or dynamical processes.

2 Structural Primitives and Enumerability

2.1 Persistence and Half-Fold Resolution

Persistence is treated here as a structural condition rather than a dynamical one, following the admissibility-based definition established in Paper II [**Rollins2026Persistence**]. A configuration persists when unresolved lattice routing is stabilized rather than dissipated. No notion of force, energy, or motion is required for this definition.

The primitive unit of unresolved routing is the *half-fold (HF)*. A half-fold represents a local asymmetry in admissible lattice continuation: one facing admits extension, while its reciprocal facing enforces return. A half-fold therefore cannot exist without simultaneously encoding its own mirror. Reflection is not imposed externally; it is intrinsic to the half-fold itself.

Persistence does not require that half-folds be resolved. It requires only that unresolved structure remain stable under lattice constraints.

Proposition 2.1 (Enumerability from Minimal Persistence)**.** *Any persistent asymmetric configuration admitted by the lattice must exhibit invariant discontinuities under admissible deformation. These discontinuities admit countability without introducing dynamics, substance, or measurement.*

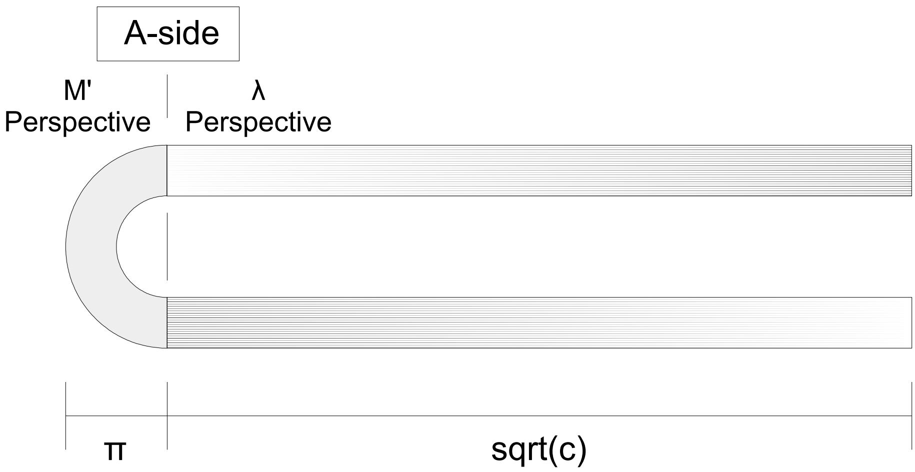

Figure 3: **Single half-fold primitive showing intrinsic reflection.** A half-fold is depicted with its forward admissible corridor and its intrinsic reflected return. The return path represents the unavoidable mirror of an unresolved fold. No directionality, force, or transport is implied; the diagram encodes only the requirement that a half-fold generates its own reciprocal facing in pursuit of closure.

Remark 2.2 ((On $\sqrt{c}$ as a half-fold corridor operator).). Throughout this paper, $\sqrt{c}$ is used as a *corridor-level operator* associated with a single half-fold. It is not interpreted here as the square-root of a measured transport speed, nor as an operation performed on an SI-valued constant. Rather, $\sqrt{c}$ names the maximal corridor potential of one half-fold, with c representing the second-order closure obtained when two compatible half-fold corridors are composed. Dimensional "faces" of these corridor forms are deferred to later interpretation papers; in Paper III they are recorded only as structural operators.

2.2 Half-Fold Countability

Once persistence is admitted, half-folds become *countable.*

Countability does not introduce discreteness by assumption; it follows from structural minimality. A persistent configuration cannot arbitrarily subdivide or merge half-folds without altering its admissible routing. As a result, the number of half-folds associated with a persistent configuration is invariant under admissible deformation.

This immediately implies that persistent configurations may differ not only qualitatively, but numerically, by half-fold inventory.

2.3 Structural Distinguishability

Two persistent configurations with different half-fold inventories cannot, in general, be deformed into one another without violating admissibility. Even configurations with equal half-fold counts may differ by internal resolution, reflection pattern, or retained structure.

This establishes *structural distinguishability.*

Structural distinguishability is not labeling, naming, or ordering. It is simply the recognition that multiple persistent configurations exist which are not mutually equivalent under admissible deformation.

From this point onward, persistence is no longer singular. It is plural.

2.4 Enumerability and Structural Indexing (Inevitability)

Once multiple inequivalent persistent configurations are admitted, *enumerability becomes unavoidable.*

Enumerability here does not imply measurement, hierarchy, or physical quantity. It is the minimal requirement that inequivalent persistent configurations be distinguishable in principle. Any framework that admits persistence together with half-fold countability must therefore admit a means of indexing persistent structures.

This paper does not yet formalize such indexing. It is sufficient at this stage to recognize that distinct persistent configurations occupy distinct structural positions within the lattice grammar.

The formal representation of this necessity will be introduced only after the relevant configurations have been established explicitly.

2.5 Admissible Seating and Structural Compatibility

Persistence alone does not determine whether two configurations may coexist.

When persistent configurations are placed in proximity, their half-fold structures may either align compatibly, restrict one another's admissibility, or prohibit stable coexistence entirely. This geometric relation is referred to as *admissible seating.*

Admissible seating does not describe interaction or force. It describes whether the unresolved structure of one configuration can be accommodated by the resolved or unresolved structure of another without violating admissibility constraints.

At this stage, admissible seating is descriptive rather than formal. However, repeated judgments of compatibility, restriction, or suppression already constitute a form of *admissible evaluation*, which will be formalized only after sufficient structural content has been established.

Evaluation of compatibility, repetition, and composite admissibility is deferred explicitly to Paper IV.

Remark 2.3 (Half-fold potential and corridor normalization).). A half-fold represents an unresolved unit of admissible propagation. As such, it cannot support a closed corridor. The maximal admissible corridor potential associated with a single half-fold is therefore the square-root of the closed corridor magnitude. We denote this maximal half-fold corridor potential by $\sqrt{c}$.

Remark 2.4. (Dual-face correspondence of the half-fold). The half-fold admits two correlational descriptions corresponding to the two structural perspectives of the codex grammar:

- On the λ face (length perspective), a single half-fold contributes corridor potential $\sqrt{c}$.

- On the M' face (inverse-length perspective), the same half-fold contributes curvature closure π.

These are not independent quantities. They are dual readings of the same structural primitive under the fixed codex correspondence. The π-quantization of the half-fold recorded in the Paper I codex is the M'-face description of the structure whose λ-face description is $\sqrt{c}$. Two half-folds in mutual resolution yield circulatory closure: $(\sqrt{c})^2 = \mathrm{c}$ on the λ face, and 2π on the M' face. The c-corridor of the hourglass grammar therefore records second-order half-fold composition.

Two compatible half-folds may resolve into a closed circulation, producing a 2π loop and restoring the full corridor magnitude c. Any retained π bookkeeping records incomplete closure of this process.

Accordingly, expressions involving powers of $\sqrt{c}$ throughout this paper record half-fold inventory and closure order, not transport speed or dimensional scaling.

2.6 Summary of Section 2

This section establishes the following, without invoking physics:

- Half-folds are the primitive units of unresolved routing.
- Persistent configurations admit invariant half-fold counts.
- Distinct persistent configurations are structurally distinguishable.
- Distinguishability implies enumerability.
- Coexistence of configurations requires admissible seating.
- Admissible seating implies the necessity of admissible evaluation.

No operators have yet been defined. Nevertheless, the structural need for both indexing and evaluation is now present. Structural consequences of persistence and closure are summarized in Appendix A.

Remark 2.5 (Why operators become inevitable.). Once half-fold inventories are countable and persistent configurations are distinguishable, a purely structural indexing becomes unavoidable. We denote this indexing operator by $\hat{Z}$. Further, once indexed structures admit coexistence and seating, an admissibility evaluator becomes unavoidable; we denote this by $\hat{\xi}$. These operators are introduced formally in Sec. 9; their necessity follows here from countability and distinguishability alone.

3 Notation, Identity, and Structural Bookkeeping

3.1 Structural Identity and Superscript Placement

This paper treats notation as grammar, not shorthand. Structural identity is determined not only by configuration, but by whether internal bookkeeping has acted. To encode this unambiguously, bookkeeping state is indicated by *superscript position*, not by operator action.

For any structural quantity x:

- **Pre-π (pure structural geometry, no retained bookkeeping):**

$$x^{\circ}$$

- **Post-π (internal phase or torsion bookkeeping accounted):**

$$^{\circ}x$$

The superscript $\circ$ does not represent an operation performed on x. Its placement indicates whether internal retention has occurred.

As a consequence, the following are generally inequivalent:

$$x^{\circ} \neq {}^{\circ}x$$

unless explicitly stated otherwise.

This grammatical distinction applies uniformly to identities, structural lengths, and composite configurations throughout the paper.

3.2 Structural Length Normalization

Structural length normalizations (λ) are introduced as descriptors of admissible scale arising from half-fold depth, closure state, and restriction of admissibility. They do not represent wavelength, energy, frequency, or motion.

As with structural identity, length normalization may appear in both pre- and post-π forms:

$$\lambda_x^{\circ} \quad \text{and} \quad {}^{\circ}\lambda_x$$

The suffix $^{\circ}$ denotes a purely structural normalization, prior to internal phase or torsion retention. The prefix $^{\circ}$ denotes that internal bookkeeping has acted and is retained.

These forms are not interchangeable. In general,

$$\lambda_x^{\circ} \neq {}^{\circ}\lambda_x$$

unless explicitly stated otherwise.

Remark 3.1. (Volumetric closure and the 4π reference). The factor 4π appearing in all structural length normalizations is the surface area of the unit sphere. It records three-dimensional volumetric closure — the minimal closed surface that distinguishes an interior structural domain in from an exterior environment. This volumetric closure is the structural precondition for persistence and for the M' face (inverse-length mass representation).

Volumetric closure (4π) is structurally distinct from circulatory closure (2π). A loop-photon frame achieves 2π closure through paired half-fold resolution but does not establish a volumetric boundary. Persistent configurations require 4π closure in addition to any circulatory structure they may contain. This distinction is the structural reason that loop photons do not persist independently: they admit circulation without volumetric closure.

3.3 Identity Without Enumeration (Yet)

At this stage of the paper, persistent configurations are structurally distinguishable but not yet formally indexed. Terms such as *electron-class*, *proton-class*, and *neutron-class* are used descriptively to indicate inequivalent persistent configurations, not to assign numerical labels or physical meaning.

The paper therefore recognizes identity without enumeration.

Enumeration becomes unavoidable only after the full set of minimal persistent configurations has been established explicitly. Until then, structural identity is treated qualitatively rather than indexically. Identity is asserted here only as distinguishability of configuration class, not as indexed enumeration. Formal indexing is deferred.

3.4 Admissible Seating Without Formal Evaluation (Yet)

Throughout the paper, persistent configurations are described as either compatible or incompatible when placed in proximity. This relation is referred to descriptively as *admissible seating.*

Admissible seating expresses whether the unresolved half-fold structure of one configuration may be accommodated by the resolved or unresolved structure of another without violating admissibility constraints. It does not imply interaction, attraction, or force.

At this stage, admissible seating is described qualitatively rather than evaluated formally. However, repeated reference to compatibility, restriction, and suppression already implies the existence of an admissibility evaluation framework, which will be introduced only after sufficient structural content has been established. No energetic, dynamical, or electromagnetic interpretation is assigned to identity at this stage.

3.5 Scope Reminder

Nothing in this section introduces dynamics, forces, or interactions. Notation serves only to prevent ambiguity as structural complexity increases.

4 Minimal Persistent Asymmetric Configuration (Electron-Class)

4.1 Structural Preconditions for Persistence

Persistence is not assumed. It must arise from admissible structural conditions within the lattice. Three requirements are necessary:

1. asymmetry, so that routing is not trivially canceled;
2. reflection, so that unresolved structure does not dissolve;
3. an admissible interface through which persistence may be stabilized.

These conditions are structural rather than dynamical and do not invoke interaction, force, or field behavior. The electron-class configuration is the minimal structure that satisfies all three simultaneously.

Lemma 4.1 (Minimal half-fold count for persistent asymmetry.)**.** *We sketch why* HF = 3 *is the minimal inventory admitting persistent asymmetry.*

1. HF = 1 **cannot persist.** A single half-fold is an unresolved interface lacking a cooperative corridor for reflection or closure. Without a complementary routing partner, its admissibility cannot be stabilized and it redistributes. Thus HF = 1 cannot support persistence.

2. HF $= 2$ **cannot support asymmetric persistence.** Two half-folds admit only two outcomes: (i) mutual closure into a symmetric reflective loop, which exhausts open inventory and yields no unresolved interface, or (ii) incomplete pairing that reduces to the HF $= 1$ case. Either way, HF $= 2$ does not admit a *persistent asymmetric* configuration.

3. HF $= 3$ **is the minimal admissible case.** With three half-folds, two may close into a reflective loop-frame while one remains unresolved. This is the first inventory that simultaneously admits (i) a stable reference closure and (ii) an open interface. Hence HF $= 3$ is minimal for persistent asymmetry.

Proposition 4.2 (Minimal Open Persistence)**.** *The minimal persistent asymmetric configuration contains exactly three half-folds, of which at least one must remain unresolved. Any configuration with fewer half-folds cannot sustain persistence, and any configuration with all half-folds resolved is internally closed.*

Proposition 4.3. *Primitive persistent configurations admitted by the lattice possess invariant half-fold inventories. Distinct inventories correspond to inequivalent closure classes and cannot be transformed into one another under admissible deformation.*

Definition 4.4 (Electron-Class Configuration)**.** An *electron-class configuration* is a minimal persistent asymmetric structure comprising three half-folds (HF), of which two resolve into a closed circulation and one remains open.

The closed pair forms a 2π loop structure, while the remaining half-fold admits redirected lattice routing and defines an open interface.

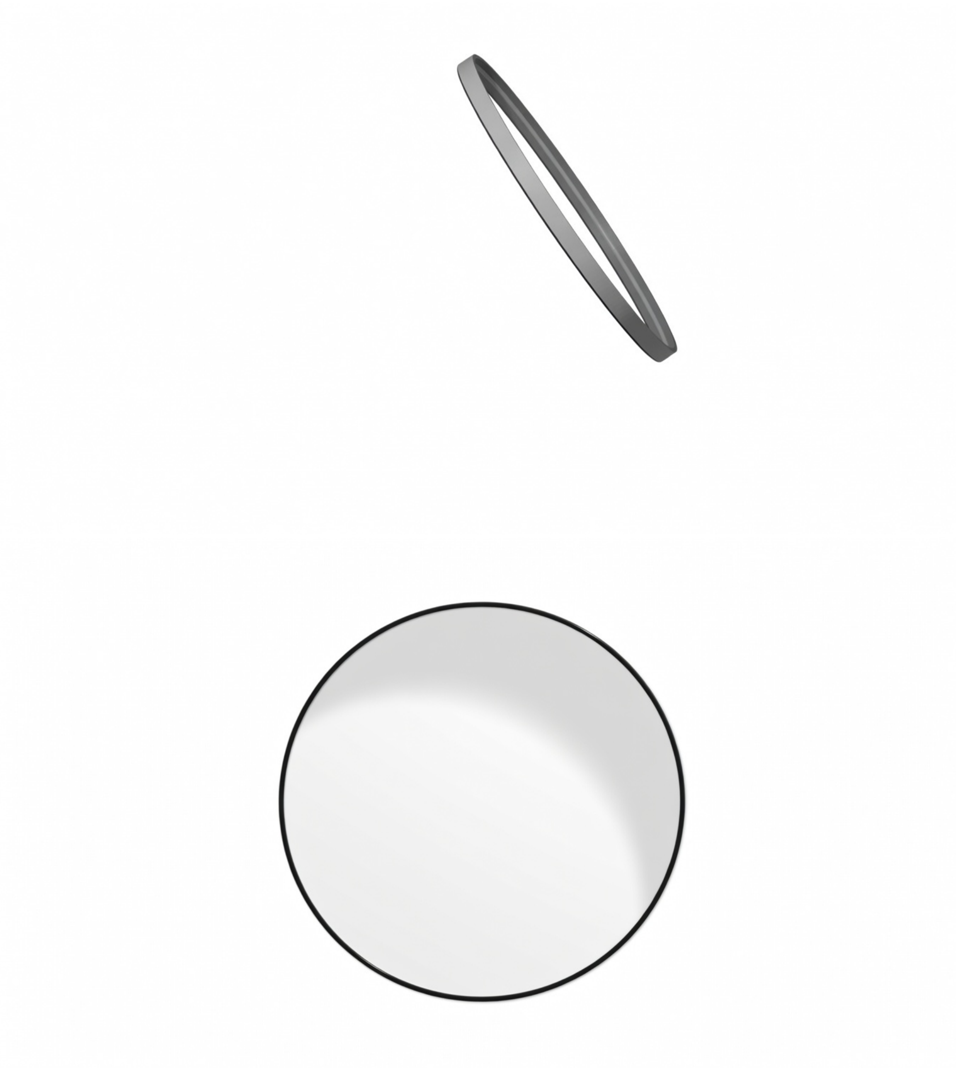

Figure 4: Electron-class configuration (representational). Upper: edge-on view showing the loop-photon frame as a two-half-fold closed circulation (2π). The ribbon structure reflects the two constituent half-folds. Lower: face-on view showing the translucent reflective plane spanning the interior of the LP frame. This plane represents the open half-fold, which occupies the structural position of mid_1 without achieving volumetric closure. The open half-fold redirects environmental admissibility and defines the configuration's charge signature. No embedding, orientation, or spatial metric is implied. The open half-fold defines an admissible routing interface rather than a geometric boundary.

4.2 Half-Fold Inventory of the Electron

The electron-class configuration contains exactly three half-folds,

$$\mathrm{HF}_e = 3. \tag{1}$$

This inventory is irreducible. Fewer half-folds cannot sustain persistent asymmetry, while greater inventories introduce closure classes belonging to distinct configurations addressed later.

The three half-folds separate into two functional roles:

- two half-folds capable of mutual reflection,
- one half-fold that remains unresolved.

Configuration	Total HF	Open HF	Loop HF	Basin HF	Torsion
Electron (e)	3	1	2	0	0
Proton (p)	4	0	0	4	0
Hydrogen (H)	7	1	2	4	0
Neutron (n)	7	$1^{\dagger}$	2	4	π

Table 1: Structural half-fold (HF) inventory of primitive and composite configurations. *Open* denotes unresolved half-folds not consumed by loop closure or basin reflection. *Loop* denotes half-folds consumed by a 2π closed circulation (loop-photon frame). *Basin* denotes half-folds consumed by proton-class reflective closure. *Torsion* records retained unresolved structure that suppresses external admissibility. †In the neutron-class configuration, an open half-fold remains present but is torsion-locked, leaving no externally admissible interface.

This distinction is geometric and topological, not directional or energetic.

Lemma 4.5 (Resolution of Paired Half-Folds)**.** *A pair of compatible half-folds that cannot remain locally admissible as unresolved structure must resolve into a closed circulation.*

Such resolution produces a 2π-closed routing unit, hereafter referred to as a loop photon. This resolution does not introduce transport, sourcing, or dissipation; it is the minimal structural closure available to paired half-folds whose continued openness is inadmissible.

Remark 4.6. When two half-folds resolve into a closed circulation, any admissible corridor potential previously associated with their unresolved configuration is repackaged into the closed loop. This loop carries no open interface and therefore admits no further redirection or persistence independently.

The appearance of a loop photon thus records the elimination of unresolved half-fold structure, not the generation of a new primitive.

4.3 Loop-Photon Frame and Reflective Organization

Two of the three half-folds may satisfy the intrinsic mirror requirement by mutual facing. When this occurs, they form a closed reflective frame with toroidal topology.

This structure is referred to as the *loop-photon frame*. The terminology denotes topology only. No propagation, emission, or field behavior is implied.

The loop-photon frame does not exhaust the electron's half-fold inventory. Its role is organizational: it establishes a reflective reference frame within which the remaining half-fold persists as an unresolved interface.

4.4 Charge as Structural Redirection

The open half-fold in the electron-class configuration does not constitute a source of lattice structure, nor does it introduce a preferred direction of transport. Its role is instead organizational: it admits redirection of ambient lattice structure relative to the loop-photon frame.

This redirection does not generate inflow, but neither does it exclude it. Rather, the presence of an unresolved half-fold conditions how lattice structure may be routed through the configuration once persistence is established. Charge, at this level, is therefore not an interaction or a carrier, but a structural signature of redirected admissibility associated with an open interface.

No assumption is made here regarding the ultimate origin or gating of lattice inflow. Only the redirection permitted by the open half-fold is asserted.

The present section establishes the structural conditions under which an external projection layer later becomes admissible; it does not yet define that layer.

4.5 Loop-Photon Circulation and Mass Correlation

The loop-photon frame provides the toroidal circulation that organizes redirected inflow. This circulation constitutes the admissible lattice engagement associated with the electron-class configuration.

Its characteristic pre-bookkeeping length λ_e° sets the stabilized persistence scale. Mass is not an independent input but correlates directly with this retained length scale.

Smaller retained length corresponds to greater stabilized lattice engagement under bookkeeping. No energetic interpretation is implied by this statement.

4.6 Geodesic Routing Modes and Destructive Loop-Photon Records

The electron-class configuration admits multiple admissible routing behaviors within the lattice. These routing modes are structural and pre-dynamical; they describe how admissibility is resolved rather than motion, transport, or force.

Mode 1 (Geodesic-Constrained Routing). In Mode 1 routing, an admissible configuration remains confined to a single geodesic. No half-fold reassignment occurs, no residual incompatibility is generated, and no structural bookkeeping is recorded. A loop-photon frame, when present, may participate in this routing mode as a non-persistent organizational structure.

Mode 2 (Geodesic-Crossing Routing). In Mode 2 routing, the electron-class configuration resolves admissibility by crossing between adjacent geodesics. Such crossings necessarily generate residual lattice structure corresponding to the geometric mismatch between the admissible paths. This residual structure closes as a two–half-fold configuration (2 HF), forming a loop photon.

Loop photons do not retain internal bookkeeping or persistence across interactions. While they may undergo further admissibility filtering, splitting, or boundary redirection, each interaction replaces rather than extends the structural record.

All photon production admitted within this framework arises from Mode 2 routing, whether through cross-geodesic admissibility of persistent configurations or through boundary filtering of existing loop photons. In all cases, the loop-photon wavelength satisfies the geometric identity $\lambda_{\mathrm{LP}} = \Delta_{\mathrm{geodesic}}$.

Remark 4.7. Remark 4.7 (Universality of Mode-2 loop-photon formation). Any admissible cross-geodesic re-routing of an electron-class configuration admits closure of the residual mismatch as a two–half-fold (2 HF) loop-photon record. Within the structural scope of Paper III, this accounts for all admitted loop-photon formation pathways (either by Mode 2 re-routing or by boundary filtering of existing loop photons).

Loop Photons as Destructive Structural Records. A loop photon is a non-persistent configuration composed of two half-folds. It records a geometric length or distance associated with admissibility reconfiguration. Because it admits lattice absorption without feedback or closure, it cannot persist indefinitely and does not support retained structural bookkeeping.

The bookkeeping carried by a loop photon is therefore *destructive*: it records structural displacement temporarily and is erased upon absorption or further reconfiguration. Loop photons do not support retained bookkeeping or independent structural normalization. Accordingly, loop photons are structurally processable but not persistently accumulative: each admissible interaction replaces the record rather than extending it.

Boundary Filtering and Secondary Loop-Photon Generation. Loop-photon generation is not limited to Mode 2 electron routing. When an incident loop photon encounters a configuration or boundary that admits only partial structural compatibility, admissibility filtering occurs. In such cases, the loop photon may be reconfigured into multiple loop photons of different structural lengths, corresponding to selective retention and disposal of admissible components.

This mechanism underlies reflection, selective absorption, and fluorescence phenomena without invoking dynamics, fields, or energetic transitions.

Remark 4.8. Boundary filtering constitutes an alternative pathway for loop-photon generation. While distinct from Mode 2 electron routing, it remains consistent with the interpretation of loop photons as destructive structural records of admissibility resolution.

Scope Limitation. This subsection establishes the structural origin of loop photons and routing modes within the electron-class configuration only. No claims are made regarding electromagnetic propagation, forces, or environmental interaction, which are deferred to later work.

Accordingly, within the structural grammar of Paper III, all photon production arises as destructive bookkeeping of admissibility reconfiguration in electron-class configurations, while loop photons themselves remain non-persistent records incapable of retained structural identity.

An interpretive supplement expanding routing modes and their phenomenological correspondence is provided separately (supplement S1).

4.7 Intrinsic Timing Without Phase Cycling

The electron-class configuration defines an intrinsic structural period arising from the relationship between the loop-photon circulation and a single admitted lattice unit,

$$t_e = \frac{\lambda_e^\circ}{c}. \tag{2}$$

This timing does not subdivide into phases and does not constitute resonance cycling. It remains singular and harmonic, and is compatible with later basin-based resonance conditions without conflict.

4.8 Bookkeeping and Electron Identity

The electron does not retain internal torsion or phase multiplicity. Consequently, the application of bookkeeping does not modify its normalization,

$$^\circ\lambda_e = \lambda_e^\circ. \tag{3}$$

Here λ_e° denotes the pre-bookkeeping geometric value, and $^\circ\lambda_e$ denotes the post-bookkeeping value. For the electron, these are equal because no retained torsion exists to be recorded.

4.9 Summary of the Electron-Class Configuration

The electron-class configuration uniquely admits multiple admissible routing behaviors without closure. In addition to single-geodesic persistence, the unresolved half-fold permits cross-geodesic reconfiguration, during which excess lattice structure is shed as nonpersistent loop-photon records. These records do not retain identity or phase, but serve as destructive bookkeeping of admissibility displacement. This capability distinguishes the electron-class from internally closed configurations and establishes the sole structural origin of photon production without invoking dynamics, forces, or fields.

The electron-class configuration is characterized by:

- a half-fold inventory of three;

- a two-half-fold loop-photon frame providing reflective organization;
- one open half-fold acting as the persistence interface;
- charge as the record of redirected routing;
- intrinsic timing $t_e = \lambda_e^\circ/c$ without phase cycling;
- trivial action of bookkeeping on its identity.

5 Internally Closed Persistent Configuration (Proton-Class)

5.1 Structural Transition from Open Interface to Basin Closure

Where the electron-class configuration admits persistence through an open half-fold supported by a reflective frame, the proton-class configuration represents the next structural transition: persistence through *internal closure*.

This transition is not additive. It does not arise by attaching an additional half-fold to the electron-class structure. Instead, it results from a reorganization of half-fold roles in which unresolved interfaces are redistributed and replaced by a central exclusion.

The defining feature of the proton-class configuration is the formation of a basin.

Proposition 5.1 (Closure by Reflection)**.** *An internally closed persistent configuration admits no unresolved half-folds and therefore suppresses charge routing. Such configurations organize admissibility through reflection, producing multiplicative closure rather than accumulation.*

5.2 Half-Fold Inventory and Basin Geometry

The proton-class configuration contains four half-folds,

$$\mathrm{HF}_p = 4. \tag{4}$$

Each half-fold terminates at a common central region that forbids through-routing. This region is not a sink, nor a source. It is an exclusion enforced by geometry.

No half-fold remains unresolved. Consequently, no persistence interface exists at the boundary of the configuration.

The proton-class configuration is therefore internally closed.

Definition 5.2 (Basin closure.)**.** A *basin* is a four-half-fold configuration with the following purely structural properties: (i) the half-folds admit pairing into two opposing corridors; (ii) through-routing across the central region is inadmissible (the corridor pairs do not admit a transmission channel); (iii) admissible routing therefore reflects across each opposing pair. A basin is thus characterized by *central exclusion* and *pairwise reflection* without invoking metric distance or curvature.

Lemma 5.3 (Minimal basin inventory.)**.** *Central exclusion requires at least two opposing corridor pairs. Therefore the minimal inventory admitting basin closure is* $\mathrm{HF} = 4$*.*

Corollary 5.4 (No loop-photon closure in the proton-class basin)**.** *In a proton-class basin configuration, the half-fold inventory is exhausted by basin closure: the admissible routing condition is satisfied by pairwise reflection across opposing half-folds under central exclusion.*

Consequently, no compatible half-fold pair remains available to resolve into an independent 2π closed circulation. Hence the proton-class configuration admits no loop photon.

Remark 5.5. A loop photon is a closure product of an *available* compatible half-fold pair; the basin closure consumes all available pairing capacity.

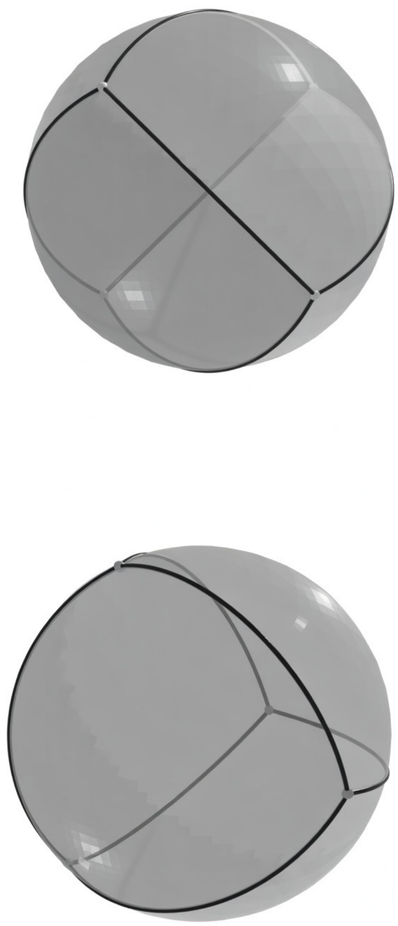

Figure 5: Proton-class basin configuration (representational). Two views of a four-half-fold internally closed basin. The four half-folds resolve into a basin structure admitting pairwise reflection under central exclusion. The surface patches and boundary lines are representational only — they indicate the partitioned closure structure of the basin without implying a specific geometric realization. Lattice interaction is purely reflective and admits no orientation, phase, or selection among facings. The three admissible pairing orientations (not shown) correspond to distinct electron seating facings and do not represent lattice traversal or internal proton dynamics. No spatial metric or embedding is implied. The depicted facings represent interface admissibility for external configurations and are not intrinsic structural states of the basin.

Corollary 5.6 (Loop-photon admission in hydrogen-class seating)**.** *In a hydrogen-class configuration, the composite half-fold inventory is not exhausted by basin-style closure alone. The configuration includes (i) a seated basin component and (ii) an additional half-fold inventory that does not participate in the basin's pairwise reflection constraint.*

Accordingly, a compatible half-fold pair remains available to resolve into an independent 2π closed circulation. Therefore the hydrogen-class configuration admits a loop photon.

All diagrams in this section depict admissible structural relations only. No diagram asserts motion, sequence, or temporal evolution.

—

5.3 Reflection as Multiplicative Admissibility

Because lattice inflow cannot pass through the basin, admissible routing is reflected. This reflection does not accumulate structure locally. Instead, reflection realizes multiplicative admissibility across opposing half-folds.

Lemma 5.7 (Second-order reflection condition).). *Each opposing corridor pair enforces a second-order closure under reflection. Composing the two opposing pairs yields a second-order condition on the composed corridor, recorded structurally as c^2. This notation records order-of-closure under reflection, not a transport law.*

Each opposing pair of half-folds contributes a second-order closure condition. Taken together, the four-half-fold basin enforces a multiplicative reflection condition recorded structurally as

$$c^2. \tag{5}$$

This quantity does not represent transport speed or stored energy. It is a resonance condition arising from geometric reflection alone.

The quantity $c^2 = \sqrt{c}^4$ records the maximal corridor potential of four half-folds composed under basin closure. It represents the full closure capacity of the proton-class configuration and serves as the lattice potential against which other configurations are structurally referenced.

The proton is therefore not a sink. It is a resonator.

Remark 5.8 (Structural Saturation of the Lepton Ladder).). A naive continuation of the torsion-indexed hierarchy suggests that successive π-retention states would scale multiplicatively without bound. However, internal normalization of the second torsion-retentive configuration reveals an over-scaling by a factor of order 4π, signaling realization of a closed surface normalization rather than the introduction of a new independent degree of freedom.

This saturation indicates that the $k = 2$ torsion-retentive configuration does not represent a freely extensible continuation of the electron-class ladder, but a structural boundary. The hierarchy therefore terminates naturally after the second π-retention, without requiring additional postulates.

5.4 Intrinsic structural cadence of the proton basin

The proton-class basin admits reflective admissibility without traversal through its interior. This supply is not continuous in a metric sense, but occurs with a characteristic structural cadence derived from the basin's pre-bookkeeping normalization. This cadence reflects closure geometry only and does not imply temporal cycling or internal phase motion.

The proton's intrinsic structural cadence is defined as

$$t_p^\circ \equiv \frac{\lambda_p^\circ}{c}. \tag{5}$$

Using the proton pre-bookkeeping length

$$\lambda_p^\circ = \frac{4\pi}{\sqrt{c}^4}, \tag{6}$$

one obtains

$$t_p^\circ = \frac{4\pi}{c}. \tag{7}$$

This quantity does not represent motion, oscillation, or an internal phase clock of the proton. It records the structural cadence at which basin reflection is available to an external open half-fold. The basin itself does not traverse phases; it admits reflection realization at a fixed structural cadence determined by its closure geometry.

Hydrogen-class seating does not arise from equality of intrinsic periods. Instead, persistence arises when the electron's intrinsic timing becomes structurally commensurate with the basin's reflection cadence under corridor routing.

Remark 5.9 (Commensurate timing without period identity). The proton basin does not share the electron's intrinsic period. The electron admits a single intrinsic timing

$$t_e^\circ = \frac{\lambda_e^\circ}{c} = \frac{4\pi}{\sqrt{c^5}},$$

while the basin admits reflection with characteristic cadence

$$t_p^\circ = \frac{\lambda_p^\circ}{c} = \frac{4\pi}{\sqrt{c^6}}.$$

These quantities are not equal. Structural commensuration arises only after corridor routing by $\sqrt{c}$, yielding the admissibility relation

$$t_e^\circ = \frac{\lambda_p^\circ}{\sqrt{c}}.$$

Hydrogen persistence is admitted from this coincidence of admissibility cadence, not from equality of periods, self-reflection, or energetic attraction.

5.5 Suppression of Charge

Because the proton-class configuration is internally closed, no open half-fold exists through which lattice inflow must be redirected.

As a result, the environmental routing recorded as charge in the electron-class configuration is unnecessary here. Reflection alone organizes admissibility.

Charge is therefore suppressed, not canceled. Its absence is structural, not neutral.

This distinction is critical: charge is not removed by compensation but rendered superfluous by closure.

—

5.6 Structural Length Normalization and Bookkeeping

The proton's pre-bookkeeping structural length is given by

$$\lambda_p^\circ = \frac{4\pi}{\sqrt{c}^4}. \tag{8}$$

Application of bookkeeping records the retained multiplicity of admissible external routing under basin closure, yielding

$$^\circ\lambda_p = \frac{4\pi \cdot 3\pi}{\sqrt{c}^4}. \tag{9}$$

Here, the bookkeeping operator $^\circ$ records retained phase structure without altering the underlying geometric scale. The appearance of the 3π factor records retained structural multiplicity under basin closure and does not correspond to internal proton states or occupation.

—

5.7 Summary of the Proton-Class Configuration

The proton-class configuration is characterized by:

- a half-fold inventory of four;
- internal basin closure forbidding through-routing;
- reflection-based multiplicative admissibility recorded as c^2;
- intrinsic reflection cadence with three-phase structure;
- suppression of charge due to absence of open interfaces;
- nontrivial action of bookkeeping on its structural normalization.

The proton-class configuration introduces resonance, timing structure, and internal closure without invoking interaction or force. It provides the structural environment within which subsequent composite and torsion-bearing configurations may arise.

6 Composite Seating Without Torsion Locking (Hydrogen-Class)

6.1 Geometric interface multiplicity of the proton basin

Definition 6.1 (External interface multiplicity of a proton-class basin)**.** Fix a proton-class basin with half-fold inventory indexed $\{1, 2, 3, 4\}$ and central exclusion, so that through-routing is inadmissible and admissible routing is realized only by local reflection across opposing half-fold pairs.

An *external interface facing* is defined as an inequivalent partition of the four external half-fold interfaces into two complementary opposing pairs available to an external open half-fold for admissible seating.

Among the six possible pairings of $\{1, 2, 3, 4\}$ into unordered pairs, only those that form complementary pairs are admissible under basin closure. These form exactly three inequivalent facing classes:

$$(12)(34), \quad (13)(24), \quad (14)(23).$$

We denote the resulting interface multiplicity by

$$\#F_p := 3.$$

6.1.1 Combinatorial Derivation of Three Facing Classes

The proton basin consists of four half-folds arranged with central exclusion. An external open half-fold seeking to seat must interface with *complementary pairs* of basin half-folds—that is, pairs whose opposing structure admits reflective compatibility.

Label the basin half-folds as $\{1, 2, 3, 4\}$. The question of how many inequivalent pairing configurations exist reduces to:

How many ways can 4 items be partitioned into 2 complementary pairs?

There are $\binom{4}{2} = 6$ ways to select the first pair:

$$\{1, 2\}, \quad \{1, 3\}, \quad \{1, 4\}, \quad \{2, 3\}, \quad \{2, 4\}, \quad \{3, 4\}.$$

However, basin seating requires **complementary opposing pairs**—selecting any pair automatically determines its complement as the pair formed by the remaining two half-folds. The six pairings therefore reduce to three equivalence classes:

$$\begin{aligned}
&\text{Select } \{1,2\} \to \text{complement } \{3,4\} \to \text{facing } (12)(34),\\
&\text{Select } \{1,3\} \to \text{complement } \{2,4\} \to \text{facing } (13)(24),\\
&\text{Select } \{1,4\} \to \text{complement } \{2,3\} \to \text{facing } (14)(23).
\end{aligned}$$

The remaining three selections are redundant:

$$\begin{aligned}
&\text{Select } \{2,3\} \to \text{complement } \{1,4\} \to \text{same as } (14)(23),\\
&\text{Select } \{2,4\} \to \text{complement } \{1,3\} \to \text{same as } (13)(24),\\
&\text{Select } \{3,4\} \to \text{complement } \{1,2\} \to \text{same as } (12)(34).
\end{aligned}$$

These three facing classes are **geometrically inequivalent** under the basin's central exclusion: no symmetry of the basin structure under central exclusion maps one facing type to another without relabeling the half-fold indices. An external open half-fold seeking admissible seating must therefore align with exactly one of these three facing types.

The interface multiplicity is therefore:

$$\#F_p = 3.$$

Remark 6.2 (Deferral to Paper IV.). The mechanism by which a seated electron's open half-fold cycles through these three facing types—the eight-step admissibility pattern and its phase structure—is addressed in Paper IV as an electromagnetic consequence of composite seating. Paper III establishes only the *geometric necessity* of three distinct facing types.

Remark 6.3 (No lattice-visible facings.). The interface multiplicity $\#F_p$ records admissibility classes for an external open half-fold only. The lattice itself does not distinguish, select, or traverse these facings. Lattice interaction with a proton-class basin remains pure local reflection across opposing half-folds and admits no orientation, phase, or choice among facings.

Remark 6.4 (Reflection at a seated interface.). When an external open half-fold becomes seated on an admissible basin-facing interface, the electron-interface reflection pathway is locally realized at that interface. This realization does not represent traversal through the basin interior. The electron's open half-fold does not generate reflection. Rather, it persists by seeking an external reflective complement. In hydrogen-class seating, the proton basin admits this reflection periodically through its intrinsic structural cadence. Persistence is admitted from the coincidence between the electron's intrinsic timing and the basin's reflection schedule, not from self-reflection or energetic attraction.

Remark 6.5 (Structural commensuration of intrinsic timings.). The electron and proton do not share an intrinsic period. The electron admits a single intrinsic timing $t_e^\circ = \lambda_e^\circ/c$, while the proton basin admits reflection with characteristic cadence $t_p^\circ = \lambda_p^\circ/c$. Structural commensuration arises only after corridor routing by $\sqrt{c}$, yielding

$$t_e^\circ = \frac{\lambda_p^\circ}{\sqrt{c}}.$$

This relation records a coincidence of admissibility cadence, not equality of periods, resonance, self-reflection, or energetic attraction.

Corollary 6.6 (Loop-photon retention in hydrogen-class seating.)**.** *The hydrogen-class configuration retains the loop-photon frame of the electron-class component. Basin seating does not correspond to a redistribution of the two-half-fold closed circulation; it constrains only the remaining open half-fold. Hydrogen therefore admits a loop-photon frame without torsion retention.*

Remark 6.7 (Structural status of basin facings.). The three basin facings derived above do not represent internal proton states, phase cycling, or dynamical alternation. They are inequivalent admissible routing classes available to an external open half-fold under basin closure.

Importantly, the existence of these facings does not depend on occupation. The proton-class configuration structurally admits three distinct routing patterns even in isolation. The multiplicity is therefore structurally available prior to occupation. It records admissible interface classes of the proton basin for an external open half-fold, without implying internal proton phase cycling, state multiplicity, or lattice-visible orientation.

—

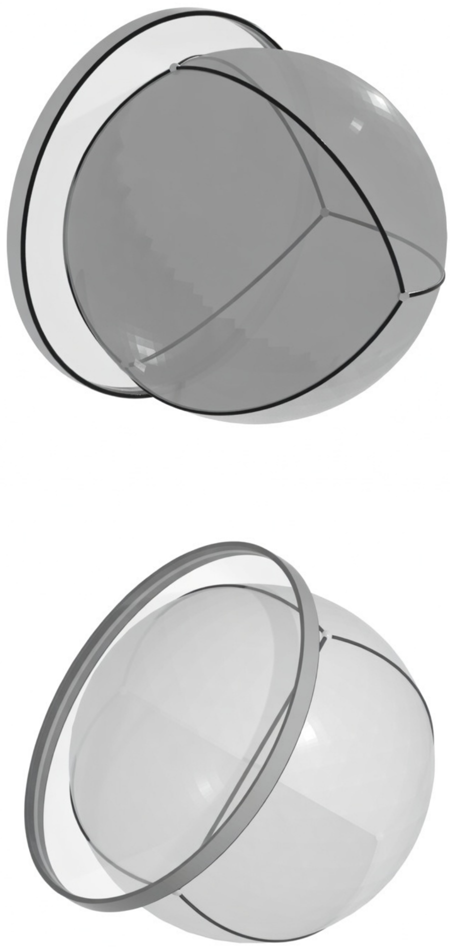

Figure 6: Hydrogen-class composite configuration (representational). Two views of an electron-class configuration seated on a proton-class basin. The loop-photon frame (ring) encircles the basin (sphere) with the open half-fold engaging one of the basin's face-pairing schedules. The translucent mirror plane of the open half-fold is visible between the ring and the basin surface, indicating that the open half-fold remains structurally unresolved — phase-restricted by the basin but not torsion-locked. The electron retains its structural identity under seating. No alternation, phase cycling, or admissibility routing is depicted. The loop-photon frame participates as overlap only and does not contribute to basin closure or support.

6.2 Hydrogen as a Support–Overlap Composite

Hydrogen is not a primitive configuration. It is an admissible composite formed by the conjunction of:

- a support-dominant structure (the proton basin), and
- an overlap-dominant structure (the seated electron-class configuration).

The proton contributes structural support through basin closure, reflection, and admissible seating interfaces. The electron contributes admissibility overlap through its unresolved open half-fold and retained loop-photon frame.

Accordingly, hydrogen is classified as a *support–overlap composite*: support is borne by the proton basin, while overlap is borne by the seated electron-class structure.

This distinction is structural only. It introduces no new primitives, corridors, operators, or dynamical claims. It records only that in hydrogen-class seating:

- support is not distributed across overlap,
- overlap is not promoted to basin support,
- admissible persistence requires both components without reducing one to the other.

Remark 6.8 (Scope of global constraints)*.* References to global admissibility constraints in this paper are purely structural. No normalization operator, scaling functional, or evaluative mechanism is introduced at this stage.

6.3 Structural Seating of the Electron

In the hydrogen-class configuration, the electron's open half-fold does not remain free. It is seated by the proton's basin environment. This seating does not disrupt the electron's intrinsic structure, nor does it convert the electron into a protonic feature.

The electron's loop-photon frame remains intact. Its open half-fold remains unresolved, but its admissible routing is now constrained by the proton basin. The electron therefore persists as an electron-class structure, but its routing environment is no longer free.

This seating suppresses free environmental redirection while preserving persistence.

Definition 6.9 (Timed partial resolution of the electron open half-fold)**.** The electron-class configuration admits an intrinsic timing

$$t_e^\circ = \frac{4\pi}{\sqrt{c}^5},$$

associated with admissible instants at which the electron open half-fold may be structurally completed by an external complementary configuration, without introducing new half-folds or retained torsion.

Lemma 6.10 (Basin-timed realization of electron reflection)**.** *A proton-class basin admits a reflection structure whose characteristic scale satisfies*

$$\lambda_p^\circ/\sqrt{c} = t_e^\circ.$$

When an electron-class configuration is seated on an admissible basin-facing interface, the basin admits reflection realization at the electron's intrinsic timing, eliminating the need for continuous self-reflection at the open interface.

Remark 6.11 (No energetic attraction)*.* The matching of electron intrinsic timing and basin reflection scale does not represent an energetic attraction or force. It is a coincidence of admissibility cadence: the basin admits structural reflection at exactly the cadence required by the electron open half-fold for persistence without self-reflection.

6.4 Structural Role Separation: Basin and Open Channel

The hydrogen-class configuration separates into two distinct structural roles:

- **Basin structure (proton-class):** provides a closed reflective environment admitting only complementary opposing half-fold routing.
- **Open-channel structure (electron-class):** carries unresolved admissibility requiring external completion.

These roles are not interchangeable. The basin does not carry unresolved admissibility, and the open channel does not define a closed reflective environment.

Hydrogen-class seating is therefore not a fusion of structures, but a constrained compatibility between:

- a structurally closed reflective basin, and
- a structurally open admissibility channel.

This separation persists across composite configurations as a distinction between closed reflective environments and open admissibility channels. No interaction, force, or field concept is introduced.

Definition. A composite configuration is structurally admissible only if basin and open-channel roles remain non-interchangeable.

—

6.5 Interface Constraint and Torsion Precondition

Definition 6.12 (π-torsion token operator)**.** Let $F = \{f_1, f_2, f_3\}$ denote the three inequivalent *interface facing classes* admitted for an external open half-fold at a proton basin. These facings are *interface admissibility classes* only: they are not intrinsic proton states and are not read by the lattice; the lattice interacts with the basin as a single reflective resonator.

Define the π-torsion token operator $\hat{\tau}_\pi$ as a *constraint on an open half-fold* that converts single-facing admissibility into simultaneous facing-lock admissibility, without changing half-fold inventory:

$$\hat{\tau}_\pi :\ \mathrm{HF}_{\mathrm{open}} \mapsto \mathrm{HF}^{(\pi)}_{\mathrm{open}} \quad \text{with} \quad \mathrm{seat}\Big(\mathrm{HF}^{(\pi)}_{\mathrm{open}}\Big) = F.$$

Equivalently, $\hat{\tau}_\pi$ is the structural constraint that binds the open half-fold to all three interface facing classes at once (a "three-facing lock"), while introducing no new corridors and no new primitive constituents.

Proposition 6.13 (Neutron as torsion-locked hydrogen seating)**.** *Let H denote the hydrogen-class seating (phase-restricted composite seating) and n the neutron-class configuration (torsion-retaining composite seating). Then the neutron-class configuration is the torsion-locked image of hydrogen-class seating:*

$$n \ \equiv\ \hat{\tau}_\pi(H),$$

in the following precise structural sense:

1. ***Inventory preservation.*** *The mapping $H \mapsto \hat{\tau}_\pi(H)$ preserves primitive inventory (no additional half-folds are introduced; no constituents are removed).*
2. ***Facing-lock entailment.*** *Hydrogen-class seating admits a single-facing restriction* $\mathrm{seat}(\mathrm{HF}_{\mathrm{open}}) = f_i$ *for some $f_i \in F$, whereas the neutron-class seat satisfies* $\mathrm{seat}(\mathrm{HF}^{(\pi)}_{\mathrm{open}}) = F$ *simultaneously.*

3. ***External charge suppression by commitment.*** *Since q' is the external routing signature of an* externally unresolved *open half-fold, the facing-lock constraint reclassifies the interface from external to internally committed, and therefore suppresses external q' realization without asserting disappearance of the open half-fold.*

4. ***Support-conditional admissibility.*** *The torsion-locked seat is admissible only under structural support provided by adjacent basin closure; without such support, the torsion-retaining configuration is structurally inadmissible.*

Sketch (structural, pre-dynamical). By construction in the hydrogen-class, the proton basin admits three inequivalent interface facing classes F for seating of an external open half-fold, and hydrogen realizes a phase restriction by seating on one facing class $f_i \in F$.

Apply $\hat{\tau}_\pi$. By Definition, $\hat{\tau}_\pi$ imposes a constraint on the same open half-fold (hence preserves inventory) whose admissible seat relation is upgraded from single-facing $\mathrm{seat}(\mathrm{HF}_{\mathrm{open}}) = f_i$ to simultaneous facing-lock $\mathrm{seat}(\mathrm{HF}^{(\pi)}_{\mathrm{open}}) = F$. This produces the neutron-class facing condition.

Because q' is defined as the externalized routing signature associated with an *externally unresolved* open interface, the three-facing lock commits the open half-fold internally across the complete facing set. Thus external q' is suppressed by structural commitment rather than by removal of the open half-fold.

Finally, the torsion-locked composite is admissible only when adjacent basin closure provides structural support compatible with the imposed constraint. In the absence of such support, the torsion-retaining configuration is structurally inadmissible. No dynamical claims (forces, motion, decay mechanisms) are required; the result is purely a change in admissibility class under the imposed constraint. □

6.6 Phase Restriction Without Closure

The defining structural feature of hydrogen is **phase restriction without torsion locking**.

The proton basin imposes admissibility constraints on the electron's intrinsic timing. These constraints restrict phase access without generating retained torsion. No additional half-folds are introduced, and no basin closure occurs beyond that already present in the proton.

As a result, the electron's intrinsic period remains defined, but its admissible phase relations are seated relative to the proton's resonance structure. Hydrogen is admitted as a seated, phase-restricted composite without torsion retention and does not define a terminal closure class.

This distinguishes hydrogen sharply from the neutron-class configuration introduced later.

6.6.1 Pre-Dynamical Consequence of Phase Restriction

Because hydrogen-class seating restricts admissible phase access without torsion locking, it does not collapse to a single terminal configuration. Instead, distinct admissibility-resolved seatings remain available under fixed support.

These distinct seatings do not alter the identity of the proton basin or the electron-class component. They alter only the admissible distribution of overlap across the available basin-facing classes.

Accordingly, hydrogen admits a discrete family of support-preserving, overlap-resolved configurations. This family provides the pre-dynamical structural basis for later observable differentiation under overlay realization, without introducing transitions, energetic levels, or dynamical emission claims into the present paper.

—

6.7 Structural Length Normalization of Hydrogen

The hydrogen-class configuration admits a characteristic structural length derived directly from the proton's pre-bookkeeping normalization:

$$\lambda_H^\circ = \frac{\lambda_p^\circ}{\sqrt{\pi}}. \tag{10}$$

Remark 6.14 (Geometric origin of the $\sqrt{\pi}$ factor). The factor $\sqrt{\pi}$ is not introduced ad hoc. The proton- and neutron-class pre-bookkeeping structural lengths satisfy $\lambda_n^\circ = \lambda_p^\circ/\pi$, separating the two closure classes by a full π of retained torsion. The hydrogen-class configuration, which is seated without torsion retention, occupies the geometric intermediate between these scales. Its structural length therefore arises as the geometric mean,

$$\lambda_H^\circ = \sqrt{\lambda_p^\circ \lambda_n^\circ} = \frac{\lambda_p^\circ}{\sqrt{\pi}}.$$

This normalization records restricted seating without retained torsion and does not define a new persistent identity.

Accordingly, no post-bookkeeping hydrogen length ${}^\circ\lambda_H$ is defined.

Remark 6.15. (Hydrogen on the torsion axis). The factor $\sqrt{\pi}$ in $\lambda_H^\circ = 4\pi/(\sqrt{\pi} \cdot \sqrt{c}^4)$ locates hydrogen at the geometric midpoint of the torsion axis between the proton and the neutron:

- Proton: $\pi^\circ = 1$ (no torsion, basin closure only)
- Hydrogen: $\pi^{1/2} = \sqrt{\pi}$ (partial constraint, phase-restricted seating without torsion lock)
- Neutron: $\pi^1 = \pi$ (full torsion lock, pseudo-closure)

The $\sqrt{pi}$ factor records the maximal torsion potential available to a seated but unlocked electron, analogous to how $\sqrt{c}$ records the maximal corridor potential of a single half-fold. Hydrogen does not select a definite torsion state; it admits a schedule of phase-restricted seatings whose characteristic scale is set by $\sqrt{\pi}$.

—

6.8 Charge Behavior in Composite Seating

In the hydrogen-class configuration, charge is neither fully suppressed nor freely expressed.

The proton basin satiates the need for charge-driven persistence. However, because the electron's open half-fold remains unresolved, charge is not annihilated. Instead, it is structurally constrained.

Charge becomes seated routing rather than free environmental routing. This distinction provides the structural precondition for later chemical behavior without invoking interaction forces. The presence of bound charge in composite seating does not introduce new photon production mechanisms beyond those established for electron-class configurations.

Remark 6.16 (No field interpretation). The distinction between bound and environmental routing is structural. It does not introduce a field, force, or transport mechanism.

—

6.9 Summary of the Hydrogen-Class Configuration

The hydrogen-class configuration is characterized by:

- no new half-folds beyond those of electron and proton;
- seating of an electron-class configuration within a proton basin;
- phase restriction without torsion locking;
- existence of a pre-bookkeeping structural length λ_H°;
- absence of a post-bookkeeping hydrogen identity;
- constrained, non-environmental charge routing.

Hydrogen therefore represents the first composite persistent structure admitted by the lattice. It marks the transition from primitive persistence to structured compositional behavior and establishes the foundation for chemistry without invoking force, interaction, or field dynamics.

7 Discrete Admissibility of Composite Configurations

The hydrogen-class configuration does not achieve internal closure. The electron remains seated across a finite set of admissible interface facings of the proton basin, with phase restriction but without torsion locking.

This constraint has a necessary structural consequence.

Admissibility is not continuous. It is constrained by:

- invariant half-fold inventory,
- finite interface compatibility (facing classes),
- and global admissibility constraints that couple the configuration as a whole.

Accordingly, the composite does not admit arbitrary structural variation. It admits only a finite set of admissible configurations.

Proposition 7.1 (Discrete Admissibility)**.** *Composite configurations under admissibility constraint form a discrete set.*

Sketch. Continuity would require deformation of admissible seating without violation of compatibility or global admissibility constraints. However, facing compatibility is finite and combinatorial, and global constraints cannot be satisfied under arbitrary variation. Therefore admissibility is discrete. □

Distinct admissible configurations correspond to distinct structural routings of the open half-fold relative to the proton basin.

Remark 7.2. This discreteness is structural. It does not arise from dynamics, oscillation, or energetic considerations.

Remark 7.3. The existence of multiple admissible configurations implies the necessity of an admissibility evaluation structure capable of distinguishing compatibility across configurations. This is precisely the role anticipated for the operator $\hat{\xi}$ in Sec. 2.6. No formal evaluation is introduced here.

Remark 7.4. When coupled to an external readout, distinctions between admissible configurations may become externally distinguishable under a readout not introduced here. The structural origin of such legibility is established here; its formal treatment is deferred.

Scope Boundary

This section introduces no dynamics, no energetic quantities, and no transition laws. It establishes only that admissibility under composite constraint is necessarily discrete.

8 Torsion Retention and the Neutron-Class Configuration

8.1 From Phase Restriction to Torsion Locking

Definition 8.1 (Structural torsion)**.** Structural torsion denotes the retention of an incompatibility in half-fold closure that cannot be resolved by admissible reflection or loop-photon formation. A torsion-retentive configuration is admissible only under continued structural support from a compatible basin environment. Torsion is recorded structurally and does not represent motion, twist, or mechanical strain.

The hydrogen-class configuration introduces phase restriction without torsion retention. A further structural transition occurs when restricted phase becomes permanently retained. This transition defines the neutron-class configuration.

The neutron is not a composite seating of electron and proton, nor does it represent a modification of the proton basin alone. It constitutes a distinct closure class characterized by torsion retention.

Proposition 8.2 (Torsion-Retained Closure)**.** *A configuration that retains phase structure under bookkeeping admits torsion locking and suppresses all admissible interfaces. Such a configuration represents maximal primitive closure.*

Corollary 8.3 (Loop-photon retention under torsion locking)**.** *The neutron-class configuration inherits the loop-photon frame present in hydrogen-class seating. Torsion locking acts only on the remaining open half-fold and does not affect the two-half-fold closed circulation. The presence of the loop photon therefore does not distinguish hydrogen from the neutron; retained torsion does.*

—

8.2 Structural Locking of the Open Half-Fold

In the neutron-class configuration, the open half-fold associated with the electron-class structure does not remain unresolved. It is structurally locked by retained torsion.

This locking does not introduce a new basin and does not generate additional half-folds. Instead, it converts an admissible open interface into a permanently constrained closure.

The distinction between restriction and locking is essential. Restriction limits admissibility without retention; locking retains phase content under bookkeeping. The neutron retains the open half-fold structurally, but that interface is no longer externally admissible once torsion lock is retained.

Definition 8.4. Restriction vs. locking. A configuration is *phase-restricted* when admissibility cycles are seated into a finite phase schedule without retaining torsion in the open inventory. A configuration is *torsion-locked* when retained torsion is recorded in the open inventory such that relaxation cannot occur without release of the retained bookkeeping. Hydrogen is treated here as restriction without locking; the neutron as locking of the open half-fold by retained torsion.

Remark 8.5 (Support requirement for torsion retention)*.* A torsion-retentive configuration is admissible only while supported by a compatible basin environment. In the absence of such support, the retained incompatibility cannot remain structurally admissible. This statement is structural only. The mechanisms by which support is maintained or lost, and the consequences of torsion release, are deferred to Paper V.

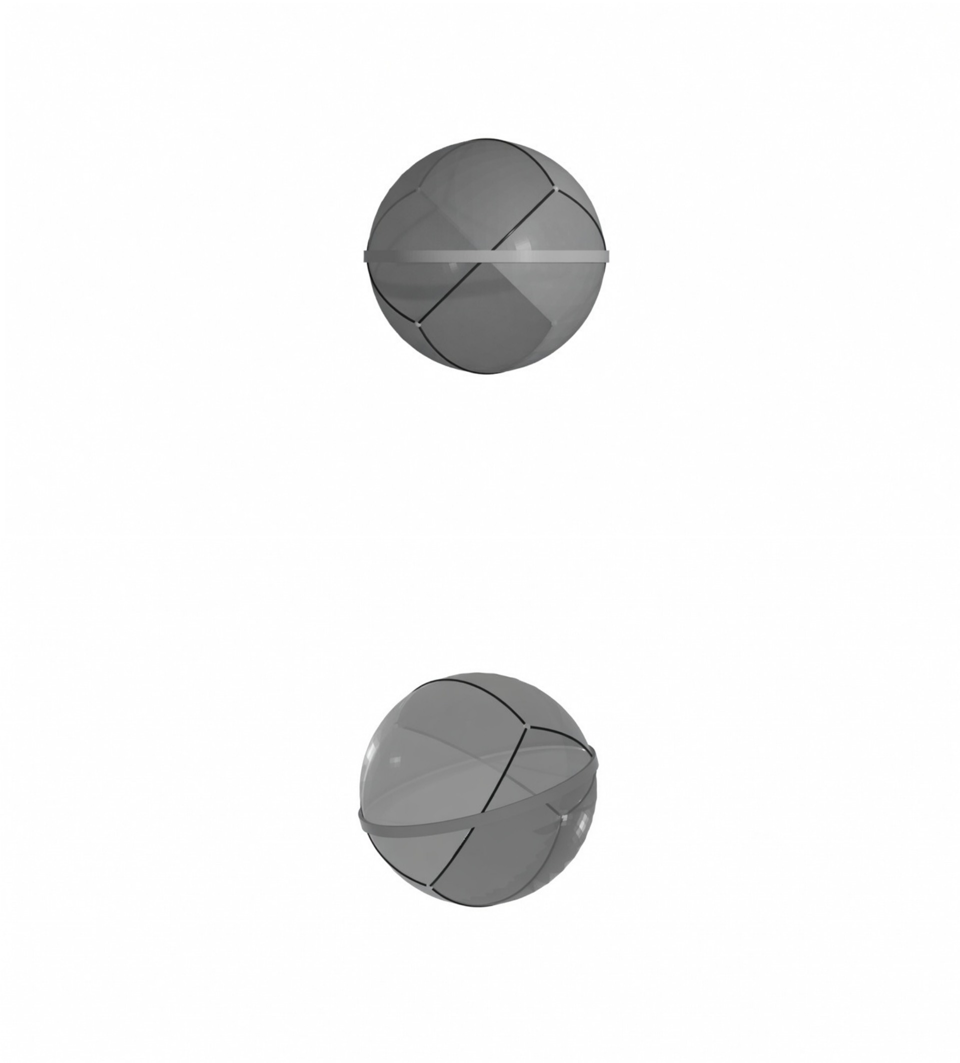

Figure 7: Neutron-class composite configuration (representational). Two views of a torsion-locked electron–proton composite. The loop-photon frame (ring) is in direct contact with the basin surface — the open half-fold's mirror plane is no longer visible because it is torsion-locked against all three proton facings simultaneously. The open half-fold remains structurally present but is not externally admissible; its routing interface has been suppressed by the torsion lock, not eliminated. No externally accessible open interface remains. The diagram depicts structural closure and bookkeeping only; it is not a claim of spatial geometry or dynamical mechanism.

Role Preservation Under Torsion Locking. Torsion locking does not convert the open-channel structure into a basin, nor does it preserve basin admissibility in its pre-seated form.

In hydrogen-class configurations, basin admissibility is resolved through seating, producing reflective saturation. This saturation corresponds to the completion of admissibility (charge satiation) rather than continued multiplicative admission.

The open-channel structure, however, does not become a basin. Its unresolved half-fold remains structurally present but is no longer externally admissible. Under torsion retention, this unresolved structure is constrained rather than resolved.

Accordingly, neutron-class configurations do not merge structural roles. The basin remains a closed reflective environment, while the open-channel persists as a suppressed admissibility channel, producing pseudo-closure rather than true saturation.

—

8.3 Torsion Retention and π-Bookkeeping

The defining feature of the neutron-class configuration is the nontrivial action of the bookkeeping operator. Retained torsion is recorded explicitly under π-bookkeeping.

The neutron's pre-bookkeeping structural length is

$$\lambda_n^{\circ} = \frac{4\pi}{\pi\sqrt{c}^4}. \tag{11}$$

Application of bookkeeping records the retained torsion, yielding

$$^{\circ}\lambda_n = \frac{4\pi \cdot 3\pi \cdot 3}{\pi\sqrt{c}^4}. \tag{12}$$

The additional π factors do not represent geometric extension. They encode retained phase content that cannot be released without structural reconfiguration. Torsion is therefore not an independent primitive, but a bookkeeping record of structurally retained incompatibility under closure.

—

8.4 Suppression of Charge and Realization of Constrained Closure Under Torsion Retention

Charge is therefore fully suppressed — not by compensation or annihilation of the open half-fold, but by structural inaccessibility of the routing interface through torsion locking. The open half-fold remains structurally present but is not externally admissible. Should torsion release occur (deferred to Paper V), the open half-fold re-emerges with its routing capacity intact.

The neutron thus represents the completion of closure within the primitive particle-class hierarchy.

—

8.5 Intrinsic Timing and Stability

The neutron-class configuration retains intrinsic timing inherited from its closure ancestry but does not introduce new phase cycling. Retained torsion stabilizes the configuration against further admissible rearrangement.

This stability does not imply permanence in the absence of structural support. It specifies only that the neutron occupies a locally maximal closure state within the lattice grammar.

Questions of decay, release of retained torsion, or coupling to other structures are deferred explicitly and do not belong to the present paper.

Quantitative treatment of unsupported torsion release is deferred to Paper V.

—

8.6 Summary of the Neutron-Class Configuration

The neutron-class configuration is characterized by:

- torsion locking of the previously open half-fold;
- nontrivial action of π-bookkeeping;
- absence of admissible open interfaces;
- full suppression of charge;
- maximal structural closure among primitive configurations.

The neutron closes the sequence of primitive persistent configurations and establishes the upper boundary of closure prior to composite and periodic structures. In the present paper, "primitive" denotes irreducible closure classes admitted by the lattice grammar prior to periodic composition; hydrogen is composite by construction, while the neutron represents a terminal closure class whose retained torsion is admissible only under supported basin association.

9 Structural Length Normalization and Bookkeeping Sequence

9.1 Pre-Bookkeeping Lengths as Structural Geometry

Each persistent configuration introduced in Sections 4–7 admits a characteristic structural length prior to bookkeeping. These pre-bookkeeping lengths encode geometric organization alone and are denoted by a right-superscript circle.

For the configurations considered here, the pre-bookkeeping structural lengths are:

$$\lambda_e^\circ = \frac{4\pi}{\sqrt{c}^3}, \tag{13}$$

$$\lambda_p^\circ = \frac{4\pi}{\sqrt{c}^4}, \tag{14}$$

$$\lambda_H^\circ = \frac{\lambda_p^\circ}{\sqrt{\pi}}, \tag{15}$$

$$\lambda_n^\circ = \frac{4\pi}{\pi\sqrt{c}^4}. \tag{16}$$

These quantities do not represent measured wavelengths or dynamical scales. They are structural normalizations that arise from half-fold inventory, closure type, and admissible reflection geometry.

All structural quantities presented in this paper are expressed in their A-side representations. Alternative representations that require explicit gravitational routing are deferred until gravity is introduced.

—

9.2 Bookkeeping as Retained Phase Recording

Bookkeeping, denoted by the left operator $^\circ(\cdot)$, records retained phase content associated with closure and torsion. Its action does not alter geometric scale; it records retained structural content carried by the configuration.

Applying bookkeeping yields:

$$^{\circ}\lambda_e = \lambda_e^{\circ}, \tag{17}$$

$$^{\circ}\lambda_p = \lambda_p^{\circ} \cdot 3\pi = \frac{4\pi \cdot 3\pi}{\sqrt{c}^{4}}, \tag{18}$$

$$^{\circ}\lambda_n = \lambda_n^{\circ} \cdot \frac{3}{\pi} = \frac{4\pi \cdot 3\pi \cdot 3}{\pi\sqrt{c}^{4}}. \tag{19}$$

For the electron, bookkeeping acts trivially because no torsion is retained. For the proton and neutron, bookkeeping records retained phase multiplicity associated with basin resonance and torsion locking, respectively.

In this sense, the neutron's $3/\pi$ factor is the capstone geometry of the single-basin build, marking the terminal compression by which the sequence culminates in neutron-class pseudo-closure.

Hydrogen does not select a single post–π configuration. Instead, it admits a spectral family of seated admissibility states produced by continuous phase exchange between proton resonance and electron seating, without torsion retention. —

9.3 Ordering of Configurations by Closure and Retention

The sequence of configurations presented in this paper is not arbitrary. It is ordered by increasing structural closure and retained content:

1. the electron-class configuration, open and asymmetrical;
2. the proton-class configuration, internally closed by reflection;
3. the hydrogen-class configuration, seated without torsion retention;
4. the neutron-class configuration, fully closed by torsion locking.

This ordering is reflected directly in the action of bookkeeping. Only configurations that retain internal phase content acquire nontrivial bookkeeping records.

—

9.4 Completion of Structural Classification

At this stage, the structural classification of persistent configurations is complete at the level of composition, seating, and retained constraint. Distinct configurations have been identified, and their admissibility behavior has been qualitatively established through the analysis of basin structure, open-channel structure, and torsion retention.

It is now evident that structural identity alone is insufficient. Configurations that are structurally distinct may differ not only in composition, but in how admissibility is expressed: whether multiplicative, constrained, or suppressed under retained structure.

This introduces two unavoidable requirements:

- a means of distinguishing structurally inequivalent configurations, and
- a means of evaluating admissibility under structural constraint.

These requirements do not arise from additional physical assumptions, but from the internal consistency of the framework itself. Once persistent configurations and composite seating are admitted, both identification and admissibility evaluation become unavoidable.

The formal introduction of these roles is deferred to the next section.

—

9.5 Summary

Structural length normalization provides a compact record of persistence geometry across primitive configurations. Pre-bookkeeping lengths encode geometric organization, while bookkeeping records retained phase content when present.

Together, they establish a complete and ordered foundation for emergence beyond primitive persistence, preparing the transition to composite and periodic structures addressed in subsequent work.

10 Inevitability of Structural Operators

10.1 Structural Breakdown Without Operators

The preceding sections establish that persistent configurations are: (i) countable by half-fold inventory, (ii) structurally distinguishable, and (iii) capable of coexistence under admissible seating.

Taken together, these conditions introduce two unavoidable structural risks:

- **Identity degeneracy**: distinct configurations cannot be stably referenced without a structural indexing mechanism.
- **Compatibility indeterminacy**: coexistence cannot be resolved without a structural criterion of admissibility.

A framework that admits distinguishable configurations but cannot consistently refer to them, or admits coexistence without determining compatibility, is structurally incomplete.

The introduction of operators is therefore not an extension of the framework, but the minimal requirement to prevent collapse into ambiguity and contradiction.

10.2 Structural Identity and the Index Operator $\hat{Z}$

Once distinguishability is admitted, identity must be preserved under reference.

Let multiple persistent configurations exist which are not equivalent under admissible deformation. Without a structural indexing mechanism, no invariant distinction between them can be maintained in statements involving more than one configuration. Distinguishability would exist in principle but fail in use.

Therefore, a mapping must exist from the set of persistent configurations to a set of distinguishable identifiers. This mapping is purely structural and carries no metric, dynamical, or hierarchical content.

We denote this mapping by the index operator $\hat{Z}$.

The operator $\hat{Z}$ assigns to each persistent configuration a unique structural identity sufficient to preserve distinguishability under reference. It does not transform configurations and does not encode physical value. Its role is purely grammatical.

Without $\hat{Z}$, identity collapses under composition of statements. With $\hat{Z}$, identity becomes structurally stable.

10.3 Admissibility and the Evaluation Operator $\hat{\xi}$

Once coexistence is admitted, compatibility must be resolved.

Given two persistent configurations placed in proximity, their combined half-fold structures may admit, restrict, or prohibit coexistence under admissibility constraints. These outcomes are determined by the structural grammar of the lattice and are not matters of interpretation.

However, without a structural criterion to determine which outcome applies, coexistence remains indeterminate. The framework would admit composite placement without resolving its validity.

Therefore, a structural evaluation must exist which determines admissibility of composite configurations. This evaluation is static and non-dynamical. It does not describe process, evolution, or interaction, but only whether a given configuration satisfies admissibility constraints.

We denote this evaluation by the admissibility operator $\hat{\xi}$.

The operator $\hat{\xi}$ evaluates structural compatibility. It introduces no force, no interaction, and no temporal ordering. It is a constraint relation on admissible structure.

Without $\hat{\xi}$, coexistence collapses into contradiction. With $\hat{\xi}$, admissible configurations are well-defined.

10.4 Operators as Structural Requirements

The operators $\hat{Z}$ and $\hat{\xi}$ are not introduced as new primitives. They are forced by the structural conditions already established:

- Countability and distinguishability force $\hat{Z}$.
- Coexistence and admissible seating force $\hat{\xi}$.

They arise prior to any dynamical, energetic, or field-theoretic interpretation. They belong to the grammar of structure itself.

No additional operators are introduced at this stage. The pair $(\hat{Z}, \hat{\xi})$ constitutes the minimal operator set required for a consistent structural framework of persistence.

10.5 Toward Structured Regularity

With $\hat{Z}$ and $\hat{\xi}$ in place, two consequences follow immediately:

1. Persistent configurations may be indexed and compared.
2. Composite admissibility may be evaluated across multiple configurations.

These conditions imply that admissible configurations occupy structured positions within an indexed space of possibilities. Constraints on admissibility do not act in isolation but propagate across compatible configurations.

The appearance of structured regularity is therefore not imposed but emerges as a consequence of indexing and admissibility.

10.6 Bridge to Subsequent Work

The present paper establishes the necessity of structural operators but does not yet formalize their action.

In subsequent work, the operators $\hat{Z}$ and $\hat{\xi}$ will be developed explicitly as:

- indexing structures over admissible configuration space,
- and admissibility relations governing composite persistence.

This development will make precise the structured regularities implied here and will show how discrete organization arises from admissibility constraints alone.

The present result is therefore a boundary statement: the grammar of persistence forces operators. The formalization of their action, and the structures that follow from it, belong to the next stage of the program.

11 Structural Support and Admissibility Overlap

11.1 Distinction

All admissible configurations decompose into two structurally distinct components: *support* and *overlap*. This distinction is primitive and must not be conflated.

11.2 Structural Support

Definition 11.1 (Structural Support)**.** Structural support is the persistence-bearing component of a configuration. It consists of nodes, basins (when present), and lattice engagement sufficient to sustain persistence.

Support is characterized by:

- closure (partial or complete),
- reflection (when applicable),
- admissible routing internal to the configuration.

Support is *locally complete*: it does not require external continuation to remain defined.

11.2.1 Properties of Support

1. **Persistence-bearing:** Support admits stable or metastable persistence.
2. **Closure-constrained:** Determined by half-fold (HF) count and closure structure.
3. **Reflection-enabled:** Basin structures enforce mirror-consistent routing.
4. **Non-transmissive:** Support does not traverse between nodes.

11.3 Admissibility Overlap

Definition 11.2 (Admissibility Overlap)**.** Admissibility overlap is the shared admissible domain between supports along connecting geodesics. It exists only relationally and does not constitute a persistence-bearing structure.

Overlap is realized along admissible geodesic connections between supports and represents shared admissibility without independent structural ownership.

11.3.1 Properties of Overlap

1. **Non-persistent:** Overlap does not sustain independent persistence.
2. **Relational:** Exists only between supports; no standalone definition.
3. **Geodesic-bound:** Confined to admissible paths connecting supports.
4. **Shared admissibility:** Represents configurations common to multiple supports.
5. **Non-closure-bearing:** Introduces no independent closure conditions.

11.4 Structural Separation

Support and overlap must remain strictly separated.

Property	**Support**	**Overlap**
Persistence	Admits persistence	Does not admit persistence
Structure	Node/basin/lattice engagement	Shared admissible domain
Closure	Defined by HF and closure	No independent closure
Locality	Locally complete	Requires multiple supports
Role	Structural carrier	Admissibility mediator

11.5 Consequences

1. **Electron and Loop Photon (LP):** Overlap-dominant configurations; not basin-supported closures.
2. **Proton and Neutron:** Support-dominant configurations; admit basin closure and persistence.
3. **Hydrogen:** Support–overlap composite:
   - proton provides support,
   - electron contributes overlap along admissible seating paths.
4. **No reduction permitted:**
   - Overlap must not be reinterpreted as support.
   - Support must not be distributed across overlap.

11.6 Codex Constraint

Any admissible configuration must specify:

- its support structure (if present),
- its overlap participation (if present).

Failure to distinguish these constitutes a violation of structural admissibility.

11.7 Non-Extension Clause

This distinction:

- introduces no new operators,
- introduces no new corridors,
- does not modify $\sqrt{c}$, $\sqrt{G'}$, or X,
- does not invoke dynamics, energy, or force.

It is a classification constraint, not a generative mechanism.

12 Conclusion

This paper has developed the emergence of persistent structure from the lattice grammar without introducing interaction, force, or dynamical law. Beginning from primitive half-fold units and their admissible combinations, a hierarchy of configurations was established in which persistence arises from geometric compatibility and constraint.

The proton-class configuration was identified as a closed reflective basin admitting structured routing under complementary opposition. The electron-class configuration was identified as an open-channel structure carrying unresolved admissibility. Their composite, the hydrogen-class configuration, was shown to preserve structural role separation while admitting constrained seating.

Further development demonstrated that admissibility may be restricted or suppressed without altering structural identity. In particular, torsion retention yields configurations exhibiting pseudo-closure, in which unresolved admissibility is retained but prevented from external expression.

The resulting framework distinguishes between structural identity and admissibility expression. Configurations are not characterized solely by their composition, but by how admissibility is admitted, constrained, or suppressed under structural constraint.

This distinction necessitates two complementary roles: indexing of structural identity and evaluation of admissibility. These are introduced formally in the subsequent section.

No appeal has been made to force, energy, probability, or field. All results arise from structural constraints inherent to the lattice grammar. The framework therefore remains predynamical, with all admissibility behavior emerging from configuration and constraint alone.

A Structural Consequences of Persistence

A.1 Half-Fold Accounting

With the introduction of explicit persistent configurations in Paper III, half-fold accounting becomes concrete. We distinguish between total half-fold inventory and usable half-fold inventory. When a loop-photon frame is present, two half-folds are consumed by that frame and do not participate independently in closure or seating.

This distinction is bookkeeping only and does not alter admissibility.

A.2 Structural Resistance Under Persistent Routing

Persistent configurations impose constraints on admissible redistribution. When closure prevents free re-routing of lattice structure, resistance to reconfiguration represents a purely structural consequence.

This resistance is not dynamical and does not correspond to force, energy, or motion. It reflects only the presence of retained structure and constrained admissibility within a persistent configuration.

A.3 Multiple Persistent Configurations

When more than one persistent configuration is present, admissible routing is no longer uniform. Structural bias arises from the simultaneous presence of multiple retained closures.

No interaction, attraction, or force is implied. The present paper assigns no interpretation to this bias beyond its structural necessity.

Persistent configurations admit three structural outcomes: reflection (middle layer), projection (Paper IV) and normalization (Paper V).

A.4 Admissibility, Support, and Structural Stability

With explicit persistent configurations introduced in Paper III, admissibility may be discussed relative to structural support without invoking temporal evolution or interaction.

Certain closure classes retain additional structure only when supported by compatible routing environments. Retention of torsion, in particular, is admissible only when closure conditions remain satisfied under basin association. When such support is absent, the torsion-retaining configuration is structurally inadmissible. Retention of torsion is admissible only under compatible closure conditions. Without the structural support provided by an adjacent basin closure, the torsion-locked configuration is not admissible.

This condition is structural rather than dynamical. No decay law, timescale, or interaction is assumed or required.

The mechanism by which unsupported torsion fails is deferred to Paper V, where the normalization corridor is introduced.

B Structural Normalizations and Metric Correspondence

This appendix summarizes the structural normalizations derived in Paper III and records their correspondence to accepted SI values. All structural quantities are defined pre-dynamically and do not invoke force, transport, or energetic mechanisms. Metric values are provided solely for scale comparison and are not used in any derivation.

Clarification on M'. The quantity M' appearing in this appendix denotes the geometric dual of λ under hourglass routing, defined structurally as $M' \equiv 1/\lambda$ within the Paper III framework. This geometric use of M' is distinct from the codex mass representation introduced in Paper I and carries no SI interpretation.

Remark B.1 (Bookkeeping propagation through derived quantities). If a structural quantity Q is defined from λ by fixed corridor or hub rules (e.g. $M' \equiv 1/\lambda$), then bookkeeping marks propagate by substitution:

$$Q^\circ := Q(\lambda^\circ), \qquad {}^\circ Q := Q({}^\circ\lambda).$$

In particular,

$$M'^\circ = \frac{1}{\lambda^\circ}, \qquad {}^\circ M' = \frac{1}{{}^\circ\lambda}.$$

Definition B.2 (Bookkeeping grammar for derived quantities)**.** Let λ denote a structural length normalization, and let $Q(\lambda)$ be any quantity defined by fixed corridor or hub rules from λ alone. The bookkeeping grammar propagates to Q by superscript position:

$$Q^\circ \equiv Q(\lambda^\circ), \qquad {}^\circ Q \equiv Q({}^\circ\lambda).$$

No additional operation is implied. The superscript position records whether internal phase or torsion bookkeeping has acted on the underlying structural quantity.

Corollary B.3 (Geometric bookkeeping of M')**.** *Within the Paper III framework, M' denotes the geometric dual of λ under hourglass routing, defined structurally as*

$$M' \equiv \frac{1}{\lambda}.$$

Accordingly, bookkeeping propagates as

$$M'^\circ = \frac{1}{\lambda^\circ}, \qquad {}^\circ M' = \frac{1}{{}^\circ\lambda}.$$

This usage is distinct from the codex mass representation $M' = m/\ell_m$ introduced in Paper I and carries no SI interpretation.

B.1 Pre-bookkeeping Structural Quantities

Entries marked "—" indicate that no single intrinsic timing is defined at the structural level for that configuration.

Configuration	λ°	M'	t°	f°
Electron (e)	$\dfrac{4\pi}{\sqrt{c}^3}$	$\dfrac{\sqrt{c}^3}{4\pi}$	$\dfrac{4\pi}{\sqrt{c}^5}$	$\dfrac{\sqrt{c}^5}{4\pi}$
Proton (p)	$\dfrac{4\pi}{\sqrt{c}^4}$	$\dfrac{\sqrt{c}^4}{4\pi}$	$\dfrac{4\pi}{c}$	$\dfrac{c}{4\pi}$
Hydrogen (H)	$\dfrac{4\pi}{\sqrt{\pi}\,\sqrt{c}^4}$	$\dfrac{\sqrt{\pi}\,\sqrt{c}^4}{4\pi}$	—	—
Neutron (n)	$\dfrac{4\pi}{\pi\,\sqrt{c}^4}$	$\dfrac{\pi\,\sqrt{c}^4}{4\pi}$	—	—

Table 2: Pre-bookkeeping structural quantities for persistent configurations. All expressions are structural. The operator $\sqrt{c}$ records single half-fold corridor potential, while π records closure or torsion bookkeeping. No metric interpretation is implied at this stage.

B.2 Post-bookkeeping Structural Quantities

Configuration	$\circ\lambda$	M'	$\circ t$	$\circ f$
Electron (e)	$\dfrac{4\pi}{\sqrt{c}^3}$	$\dfrac{\sqrt{c}^3}{4\pi}$	$\dfrac{4\pi}{\sqrt{c}^5}$	$\dfrac{\sqrt{c}^5}{4\pi}$
Proton (p)	$\dfrac{4\pi \cdot 3\pi}{\sqrt{c}^4}$	$\dfrac{\sqrt{c}^4}{4\pi \cdot 3\pi}$	—	—
Neutron (n)	$\dfrac{4\pi \cdot 3\pi \cdot 3}{\pi\,\sqrt{c}^4}$	$\dfrac{\pi\,\sqrt{c}^4}{4\pi \cdot 3\pi \cdot 3}$	—	—

Table 3: Post-bookkeeping structural quantities. Additional π factors record retained phase multiplicity or torsion locking. No new primitive structure is introduced by bookkeeping.

B.3 Structural Length Summary

This provides a consolidated summary of all structural length normalizations introduced in Paper III. These expressions are purely structural and arise from half-fold inventory, closure geometry, and retained bookkeeping. No dynamical, energetic, or field interpretation is implied.

Configuration	λ° (pre-π)	$\circ\lambda$ (post-π)	Notes
Electron (e)	$\frac{4\pi}{\sqrt{c}^3}$	$\frac{4\pi}{\sqrt{c}^3}$	Trivial bookkeeping
Proton (p)	$\frac{4\pi}{\sqrt{c}^4}$	$\frac{4\pi \cdot 3\pi}{\sqrt{c}^4}$	Three-phase basin resonance
Hydrogen (H)	$\frac{4\pi}{\sqrt{\pi}\sqrt{c}^4}$	—	No post-π identity
Neutron (n)	$\frac{4\pi}{\pi\sqrt{c}^4}$	$\frac{4\pi \cdot 3\pi \cdot 3}{\pi\sqrt{c}^4}$	Torsion retention

Table 4: Summary of structural length normalizations for all configurations introduced in Paper III. All expressions arise from half-fold inventory and closure geometry. π appears only as retained bookkeeping. The post-π factors for the proton and neutron record retained structural multiplicity and torsion locking, respectively. They do not imply internal phase cycling or temporal alternation.

B.4 Reference SI Values

Reference SI values corresponding to the structural quantities listed above. These values are provided for scale comparison only and are not used in any derivation.

Configuration	λ (m)	Mass (kg)	Period (s)	Frequency (Hz)
Electron (e)	2.426×10^{-12}	9.109×10^{-31}	8.09×10^{-21}	1.24×10^{20}
Proton (p)	1.321×10^{-15}	1.673×10^{-27}	4.40×10^{-24}	2.27×10^{23}
Neutron (n)	1.319×10^{-15}	1.675×10^{-27}	4.40×10^{-24}	2.27×10^{23}

Table 5: All structural quantities shown correspond numerically to accepted SI values at the sub-percent level after bookkeeping. No fitting, calibration, or empirical input is performed.

B.5 Extended Half-Fold Accounting

This appendix summarizes half-fold (HF) allocation across primitive and composite configurations introduced in Paper III. The accounting distinguishes between total inventory and functional roles under admissible closure.

Configuration	Total HF	Open HF	Loop HF	Basin HF	Torsion
Electron (e)	3	1	2	0	0
Proton (p)	4	0	0	4	0
Hydrogen (H)	7	1	2	4	0
Neutron (n)	7	$1^{\dagger}$	2	4	π

Table 6: Structural half-fold (HF) inventory. *Open* denotes unresolved half-folds not consumed by loop closure or basin reflection. *Loop* denotes half-folds consumed by a 2π closed circulation (loop-photon frame). *Basin* denotes half-folds consumed by proton-class reflective closure. *Torsion* records retained incompatibility that suppresses external admissibility. †In the neutron-class configuration the open half-fold remains structurally unresolved but is torsion-locked and therefore not externally admissible.

An open half-fold denotes unresolved admissible routing not consumed by loop closure or basin reflection. In the neutron-class configuration, the remaining open half-fold is torsion-locked and therefore not externally admissible.

C Notation and Structural Grammar Index

This appendix summarizes notation introduced in Paper III for reference only. No new definitions are introduced.

C.1 Superscript Grammar

- x° — pre-bookkeeping structural quantity
- ${}^\circ x$ — post-bookkeeping (retained phase or torsion)
- $x^\circ \neq \circ x$ unless stated

C.2 Structural Quantities

- HF — half-fold count
- λ° — pre-bookkeeping structural length
- ${}^\circ\lambda$ — post-bookkeeping length
- $\sqrt{c}$ — single half-fold corridor potential

C.3 Operators

- $\hat{Z}$ — structural index operator
- $\hat{\xi}$ — admissibility evaluator
- $^\circ$ — bookkeeping operator

C.4 Key structural claims (one-line index)

- Minimal persistent asymmetry requires HF $= 3$ (Lemma 4.1).
- Proton-class basin is minimal reflective closure with HF $= 4$ (Lemma 5.3).
- Basin closure admits no loop-photon frame (Cor. 5.4).
- Proton basin admits three inequivalent external facing classes, $\#F_p = 3$ (Def. 6.1; Sec. 6.1.1).
- Hydrogen is seating with phase restriction but no torsion locking (Sec. 6.3).
- Neutron is torsion locking of the remaining open half-fold; torsion retention requires support (Def. 7.1; Rem. 7.5).

References

[1] Rollins, J. Length–Mass Reduction (LMR) Theory: Paper I — Codex and Foundational Grammar. Zenodo, 2025. doi:10.5281/zenodo.18112473

[2] Rollins, J. Length–Mass Reduction (LMR) Theory: Paper II — Lattice, Perturbation, and Persistence. Zenodo, 2026. doi:10.5281/zenodo.18178159

How to Cite This Paper

If you wish to cite this work, please use the following reference:

> Rollins, J. Length–Mass Reduction (LMR) Theory: Paper III — Emergence and Structure. manuscript, 2026.

If referring to specific structural results or definitions, please cite the corresponding paper in the LMR series (Paper I or Paper II) in addition to this manuscript. This paper is part of a structured sequence; definitions, notation, and structural grammar established in earlier papers are assumed unless explicitly redefined.