Companion Volume I

Overlay Architecture

Non-Normative Companion to the LMR Predynamical Codex

Length–Mass Reduction (LMR) Theory

Jacob Rollins

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Reading Guide: Structural Grammar, Overlay, and Dimensional Correlation

Purpose of This Guide

The LMR framework is organized into three formally distinct tiers. These tiers are independent in role, scope, and authority. Confusion between tiers leads to category error.

This guide specifies:

- What each tier does,
- What each tier does not do,
- How information flows between tiers,
- What is prohibited from flowing between tiers.

No physical claim should be interpreted outside its proper tier.

Tier 1 — Structural Grammar (Papers I–V)

Tier 1 defines the predynamical structural grammar of LMR.

It establishes:

- Corridor primitives,
- Admissibility conditions,
- Closure structure,
- Torsion engagement,
- Class identity.

Tier 1 is:

- Pre-dynamical,
- Time-independent,
- Non-probabilistic,
- Non-statistical,
- Ontologically declarative.

Tier 1 does not:

- Introduce evolution equations,
- Introduce hazard functions,
- Introduce transport equations,
- Introduce stochastic mechanisms,

- Perform dimensional reduction on SI,
- Appeal to empirical fitting.

Structural grammar determines admissibility. It does not determine dynamics.

Tier 2 — Overlay Layer (Companion Volume I)

Tier 2 introduces declared representational overlays.

An overlay is a formally stated mapping from structural scale parameters to observer-side equations.

Tier 2:

- Operates only on structural quantities already defined in Tier 1,
- Introduces Model Rules (M#),
- Introduces Dynamical Postulates (D#),
- Applies Structural Constraints (SC#),
- Derives observer-side equation families under explicit rule declarations.

Tier 2 does not:

- Modify structural grammar,
- Introduce new structural primitives,
- Claim that any admitted equation is structurally inevitable,
- Promote representational equations to ontological status.

Time appears in Tier 2 only through declared update postulates. No time parameter exists in Tier 1.

Multiple distinct equation families may arise from the same structural cadence scale depending on declared update symmetry.

This non-uniqueness is intentional.

Tier 3 — Dimensional Correlation Layer (Companion Volume II)

Tier 3 performs lm-reduction on standard SI equations.

Its purpose is dimensional translation and representational alignment.

Tier 3:

- Eliminates explicit kilogram dependence,
- Introduces the bridge quantity $\ell_m = h/c$,
- Rewrites SI equations in kg-free form,
- Preserves algebraic structure exactly,
- Introduces no new physics.

Tier 3 does not:

- Assert structural primitives,
- Assert corridor operators,
- Assert admissibility claims,
- Introduce dynamical postulates,
- Derive new empirical predictions,

- Modify established theory.

Tier 3 performs representational alignment only.
Interpretive conclusions cannot arise from dimensional reduction alone.

Inter-Tier Discipline

The tiers are orthogonal.

- Tier 1 does not imply Tier 2.
- Tier 2 does not modify Tier 1.
- Tier 3 does not justify Tier 1.
- Tier 3 does not derive Tier 2.

Permitted flow:

- Tier 1 supplies structural quantities to Tier 2.
- Tier 1 supplies dimensional primitives to Tier 3.
- Tier 2 may be dimensionally expressed through Tier 3.

Prohibited flow:

- No overlay equation may be used to infer structural ontology.
- No dimensional re-expression may be used to assert structural necessity.
- No SI algebra may be used to define structural primitives.

How to Read This Work

When encountering an equation, ask:

1. Is this a structural statement? (Tier 1)
2. Is this an overlay postulate? (Tier 2)
3. Is this a dimensional re-expression of SI? (Tier 3)

Interpret the equation only within its tier.
Cross-tier inference without explicit declaration is invalid.

Summary

LMR consists of:

- A predynamical structural grammar,
- A declared overlay layer admitting representational evolution equations,
- A dimensional correlation layer eliminating kilogram dependence.

These tiers must remain formally separated for the framework to remain coherent.

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Companion Volume I: Overlay Layer

Preface

This companion volume belongs to the Length–Mass Reduction (LMR) program.

Papers I–V establish the predynamical structural grammar of LMR. They define primitives, admissibility conditions, corridor operators, torsion engagement, and class identity. They introduce no evolution equations.

The present volume introduces declared realization overlays operating on structural cadence scales. All observer-side equations admitted herein arise from explicitly declared overlay rules. No equation is presented as structurally inevitable.

A subsequent companion volume performs lm-reduction on standard SI equations. That procedure eliminates explicit kilogram dependence while preserving algebraic structure exactly. It introduces no structural primitives and no dynamical postulates.

The tiers remain formally separated:

- Tier 1 — Structural Grammar
- Tier 2 — Overlay Layer
- Tier 3 — Dimensional Correlation

Cross-tier inference without explicit declaration is invalid.

Chapter 1

Orientation and Overlay Discipline

1.1 Structural Grammar and Representational Overlays

Papers I–V establish the structural grammar of the Length–Mass Reduction (LMR) framework. They define corridor primitives, admissibility conditions, torsion engagement, closure conditions, admissibility structure, and class identities. They do not introduce dynamical evolution laws.

The present volume does not modify that grammar.

Instead, it introduces declared representational overlays that operate on structural cadence scales already defined. These overlays are constructed through explicitly stated Model Rules (M#), Dynamical Postulates (D#), and Structural Constraints (SC#).

No structural primitive is added. No admissibility condition is altered. No geometric claim is introduced.

1.2 What an Overlay Is

An overlay, in this volume, is a declared mapping from structural scale parameters to observer-side equations.

An overlay consists of:

1. A structural input (e.g., cadence scale),
2. A declared rule relating that scale to a representational variable,
3. A stipulated update condition (memoryless, symmetric, persistent, etc.),
4. The resulting observer-side equation admitted under those declarations.

The resulting equations are not structural necessities. They are representational forms consistent with the declared overlay assumptions.

1.3 No Ontological Extension

Nothing in this volume asserts that:

- diffusion is structurally inevitable,
- waves are geometric consequences of torsion,
- hazard functions are intrinsic to admissibility classes,
- probabilistic interpretation is required by structural grammar.

All such forms arise only under explicitly stated overlay rules.

Structural grammar remains pre-dynamical and time-independent. Overlays are representational constructions applied to that grammar.

1.4 Cadence as Structural Scale

Structural cadence, as defined in the preceding papers, is treated here strictly as a scale parameter.

In this volume, cadence does not act. It does not generate. It does not propagate.

It is admitted into overlay rules as a dimensional scale from which observer-side coefficients may be constructed.

Structural cadence functions solely as a dimensional scale input.

The structural grammar does not privilege a unique dynamical realization.

Distinct equation families arise exclusively from distinct declared update assumptions (M#, D#), not from cadence itself.

Non-uniqueness is therefore an overlay property, not a structural ambiguity.

This non-uniqueness is an inherent property of the overlay layer.

1.5 Declared Rule Hierarchy

For clarity, the following hierarchy is enforced throughout:

- **Model Rules (M#):** Structural-to-representational mappings.
- **Dynamical Postulates (D#):** Explicitly declared update stipulations.
- **Structural Constraints (SC#):** Bounds inherited from corridor definitions.

Each observer-side equation in this volume is traceable to a specific combination of M#, D#, and SC# declarations.

No equation is presented without such traceability.

1.6 Interpretive Restraint

The equations derived in this volume are observer-side overlays admitted under declared rules.

They are not promoted to ontological status.

Interpretive conclusions, if any, must arise from structural grammar (Papers I–V) or from clearly stated overlay assumptions, not from equation form alone.

Hierarchy Reminder

Chapter 1 Recall

- Structural grammar remains unchanged.
- Overlays are declared representational mappings.
- Cadence functions only as a scale parameter.
- All equations depend on explicit M#, D#, SC# rules.
- No ontological extension is introduced.

Chapter 2

Overlay Rule Registry

2.1 Purpose of This Chapter

This chapter declares the representational rule registry used throughout Companion Volume I.

No observer-side equation is derived in this chapter. No probabilistic interpretation is asserted. No force, field, transport mechanism, or dynamical law is introduced at the structural level.

The purpose of this chapter is to specify the rule classes by which Tier 1 structural quantities may be admitted into Tier 2 overlay equations.

The rule registry is divided into five classes:

1. Structural Inputs (SI#),
2. Scale-Mapping Rules (MS#),
3. Algebraic-Mapping Rules (MA#),
4. Observer-Side Update Postulates (D#),
5. Structural Constraints (SC#).

Each observer-side equation admitted later in this volume must carry an explicit traceability certificate identifying the Structural Inputs, Scale-Mapping Rules, Algebraic-Mapping Rules, Observer-Side Update Postulates, and Structural Constraints under which it is admitted.

No equation may be introduced without such traceability.

2.2 Tier 2 Rule Classes

Tier 2 does not derive dynamics from structural grammar.

Tier 2 declares representational overlays over structural quantities already defined in Tier 1. The equation families obtained in Tier 2 depend on the declared overlay rules, not on structural grammar alone.

The rule classes used in this volume are defined as follows.

Structural Inputs (SI#)

Structural Inputs identify which Tier 1 quantities are admitted into the overlay.

They do not introduce new primitives.

They do not modify the Tier 1 codex.

They do not select an observer-side equation family by themselves.

Scale-Mapping Rules (MS#)

Scale-Mapping Rules declare how admitted Tier 1 scale quantities are represented as observer-side coefficients, step sizes, intervals, or parameters.

Scale maps are declared correspondences. They are not structural necessities.

Algebraic-Mapping Rules (MA#)

Algebraic-Mapping Rules declare how Tier 1 admissibility relations, indexing relations, or structural adjacency relations are represented as observer-side algebraic relations.

Algebraic maps specify representational form only. They do not assert motion, transfer, transport, interaction, or physical exchange.

Observer-Side Update Postulates (D#)

Observer-Side Update Postulates declare how a representational quantity is allowed to update inside a chosen overlay.

The label *D#* is retained because these postulates govern observer-side dynamical form. They are not Tier 1 dynamical laws.

A *D#* postulate may introduce memoryless update, symmetric update, retained-increment update, or other explicitly declared update structure.

No *D#* postulate may be read backward into Tier 1.

Structural Constraints (SC#)

Structural Constraints apply bounds inherited from Tier 1 corridor discipline or admissibility structure to the observer-side overlay.

They restrict admissible overlay parameters.

They do not introduce new structural primitives or new Tier 1 laws.

2.3 Structural Inputs (SI#)

SI1 — Structural Cadence Scale

A structural cadence scale already defined in Papers I–V may be admitted as an input to a Tier 2 overlay.

The cadence scale is treated purely as a dimensional scale parameter.

It does not act.

It does not generate.

It does not propagate.

It does not select an equation family.

The cadence scale may enter an overlay only through an explicitly declared Scale-Mapping Rule.

SI2 — Structural Length Scale

A structural length scale already defined in Papers I–V may be admitted as an input to a Tier 2 overlay.

The length scale is treated as a representational scale available for step-size declaration.

It is not interpreted as a path, trajectory, displacement, transport distance, or physical medium.

The length scale may enter an overlay only through an explicitly declared Scale-Mapping Rule.

SI3 — Structural Index Class

A structural admissibility class, lattice index, facing class, seating class, or other Tier 1 indexing relation may be admitted as an input to a Tier 2 overlay.

Such an index identifies representational distinguishability only.

It does not assert motion between indices.

It does not assert transition, occupation, probability, or exchange.

Any algebraic relation among such indices must be declared through an Algebraic-Mapping Rule.

2.4 Scale-Mapping Rules (MS#)

MS1 — Cadence-to-Coefficient Map

A structural cadence scale f_0 may be mapped to an observer-side coefficient.

The general declared form is

$$\kappa := a f_0,$$

where a is a dimensionless overlay parameter.

The coefficient κ is representational only.

It is not a force, field, generator, rate law, structural primitive, or Tier 1 mechanism.

This mapping is declared, not derived.

MS2 — Scale-to-Step Identification

A structural length scale λ_0 and cadence scale f_0 may be identified with an observer-side step size and update interval.

The general declared form is

$$\Delta x := \alpha \lambda_0, \qquad \Delta t := \frac{\beta}{f_0},$$

where α and β are dimensionless overlay parameters.

The quantities Δx and Δt are observer-side representational steps. They are not Tier 1 spatial displacement or Tier 1 time evolution.

This identification is declared, not geometrically forced.

MS3 — Derived Velocity Parameter

When MS2 is invoked, an observer-side velocity parameter may be defined by

$$v := \frac{\Delta x}{\Delta t}.$$

Using the declared MS2 form,

$$v = \frac{\alpha}{\beta} \lambda_0 f_0.$$

The parameter v is an overlay coefficient.

It is not a Tier 1 propagation speed.

It is not a structural mechanism.

It is not promoted to Tier 1 by this volume.

Any attempt to identify an observer-side velocity parameter with a structural corridor quantity would require a separate, explicitly declared work outside the scope of Companion Volume I and could not be inferred from Tier 2 alone.

MS4 — Diffusion-Coefficient Map

When MS2 is invoked together with a symmetric adjacency update, an observer-side diffusion coefficient may be defined by

$$D := \frac{\Delta x^2}{2\Delta t}.$$

Using the declared MS2 form,

$$D = \frac{\alpha^2}{2\beta}\lambda_0^2 f_0.$$

The coefficient D is representational only.

The dimensionless parameters α and β remain free overlay parameters unless a specific overlay declares otherwise.

The simplest normalized overlay choice,

$$\alpha = 1, \qquad \beta = 1,$$

gives

$$D = \frac{\lambda_0^2 f_0}{2}.$$

This normalized choice is not structurally required. It is a representational convention available within a declared overlay.

It is not a physical diffusivity unless an external interpretation is separately declared outside Tier 1.

MS5 — Persistence-Parameter Declaration

A finite persistence parameter γ may be introduced to distinguish memoryless update from retained-increment update.

The parameter γ is observer-side only.

It is not a Tier 1 persistence primitive.

It is not a structural support condition.

It is not a force, drag term, loss mechanism, or energetic dissipation.

Its role is to parameterize the damping of retained update structure inside a declared overlay.

2.5 Algebraic-Mapping Rules (MA#)

MA1 — Discrete Adjacency Mapping

A representational index n may be declared to admit adjacent indices $n-1$ and $n+1$.

This rule permits algebraic expressions involving nearest-neighbor index relations.

It does not assert motion from one index to another.

It does not assert transfer, transport, exchange, probability flow, or physical interaction.

It specifies only that the overlay representation may use adjacent-index algebra.

MA2 — Second-Difference Mapping

When MA1 is invoked, the centered second difference may be used:

$$\delta^2 P_n := P_{n+1} - 2P_n + P_{n-1}.$$

This algebraic structure represents adjacency curvature in the observer-side index variable.

It does not assert geometric curvature, field curvature, force, or physical bending at the Tier 1 level.

2.6 Observer-Side Update Postulates (D#)

D1 — Memoryless Proportional Update

A scalar observer-side quantity $S(t)$ may be declared to update in proportion to its present value only.

The declared form is

$$\frac{dS}{dt} = -\kappa S.$$

This postulate introduces memoryless proportional update.

It does not assert structural decay.

It does not introduce a hazard mechanism.

It does not assert that rarefaction, loss, or relaxation is ontologically present in Tier 1.

D2 — Symmetric Adjacency Update

When MA1 is invoked, adjacent-index contributions may be declared symmetric.

The discrete declared form is

$$P_n^{\mathrm{new}} = \frac{1}{2}\left(P_{n-1} + P_{n+1}\right).$$

Equivalently, the update increment is

$$\Delta P_n = P_n^{\mathrm{new}} - P_n = \frac{1}{2}\left(P_{n-1} - 2P_n + P_{n+1}\right).$$

This postulate declares symmetric redistribution over adjacent representational indices.

It does not assert physical diffusion, motion, transport, or exchange.

D3 — Retained-Increment Update

A representational update may be declared to retain dependence on the previous update increment.

In discrete form, retained-increment structure is represented by the second time difference

$$\frac{P_n(t+\Delta t) - 2P_n(t) + P_n(t-\Delta t)}{\Delta t^2}.$$

This postulate is the source of second-order observer-side update structure.

It does not assert Tier 1 inertia.
It does not assert structural memory.
It does not introduce a dynamical law into the codex.
It declares only that the observer-side overlay retains increment history.

D4 — Linear Retained-Update Damping

When D3 is invoked, a linear damping term may be declared using the persistence parameter γ:

$$2\gamma\frac{P_n(t) - P_n(t-\Delta t)}{\Delta t}.$$

This postulate controls attenuation of retained-increment structure within the observer-side overlay.

It does not assert physical friction, loss, resistance, or dissipation at the Tier 1 level.

2.7 Structural Constraints (SC#)

SC1 — Finite Corridor Bound

Any observer-side velocity parameter v introduced under MS3 must satisfy the inherited corridor bound

$$v \leq c.$$

This bound is inherited from Tier 1 corridor discipline.
It constrains the observer-side overlay.
It does not turn c into a Tier 1 propagation mechanism.

SC2 — No Ontological Promotion

No observer-side coefficient, step, interval, update rule, or equation family may be promoted to Tier 1 ontology.

In particular:

- κ is not a structural decay rate.
- D is not a structural diffusivity.
- v is not a structural propagation mechanism.
- γ is not a Tier 1 persistence primitive.
- $P_n(t)$, $S(t)$, and related functions are observer-side representational quantities.

SC3 — No Backward Inference

No equation admitted under Tier 2 may be used to infer a new Tier 1 primitive, corridor, admissibility condition, or structural necessity.

Overlay equations may be consistent with Tier 1 structure.
They do not define Tier 1 structure.

2.8 Traceability Certificate Requirement

Every observer-side equation admitted in this volume must be accompanied by a traceability certificate.

The certificate must identify:

1. the admitted Structural Inputs,
2. the Scale-Mapping Rules invoked,
3. the Algebraic-Mapping Rules invoked,
4. the Observer-Side Update Postulates invoked,
5. the Structural Constraints applied,
6. the resulting observer-side equation,
7. the boundary statement preventing ontological promotion.

The required certificate form is:

$$\boxed{\mathrm{SI} + \mathrm{MS} + \mathrm{MA} + \mathrm{D} + \mathrm{SC} \Longrightarrow \text{Admitted Observer-Side Form}}$$

A chapter may omit an unused category only by explicitly writing “none invoked.”

For example, a memoryless scalar overlay may have no Algebraic-Mapping Rule. In that case, its certificate must state:

$$\mathrm{MA} : \mathrm{none invoked}.$$

No equation may appear without this certificate.

2.9 Rule-Use Prohibitions

The following prohibitions apply throughout Companion Volume I.

1. No Scale-Mapping Rule may be treated as structurally inevitable.
2. No Algebraic-Mapping Rule may be interpreted as physical motion, transport, exchange, propagation, or interaction.
3. No Observer-Side Update Postulate may be read backward into Tier 1.
4. No Structural Constraint may be used to introduce a new structural primitive.
5. No observer-side equation may be used to modify the codex established in Papers I–V.
6. No overlay coefficient may be interpreted as a physical quantity unless an external correspondence layer is explicitly declared.
7. No continuum limit may be treated as a Tier 1 operation.
8. No equation family may be presented as the unique dynamical realization of a structural cadence scale.

2.10 Registry Summary

The rule registry used in the following chapters is summarized as follows.

Structural Inputs

- SI1 — Structural Cadence Scale.
- SI2 — Structural Length Scale.
- SI3 — Structural Index Class.

Scale-Mapping Rules

- MS1 — Cadence-to-Coefficient Map.
- MS2 — Scale-to-Step Identification.
- MS3 — Derived Velocity Parameter.
- MS4 — Diffusion-Coefficient Map.
- MS5 — Persistence-Parameter Declaration.

Algebraic-Mapping Rules

- MA1 — Discrete Adjacency Mapping.
- MA2 — Second-Difference Mapping.

Observer-Side Update Postulates

- D1 — Memoryless Proportional Update.
- D2 — Symmetric Adjacency Update.
- D3 — Retained-Increment Update.
- D4 — Linear Retained-Update Damping.

Structural Constraints

- SC1 — Finite Corridor Bound.
- SC2 — No Ontological Promotion.
- SC3 — No Backward Inference.

Hierarchy Reminder

Chapter 2 Recall

- Tier 2 admits observer-side overlays only through declared rules.
- Structural Inputs identify which Tier 1 quantities may enter an overlay.
- Scale-Mapping Rules determine how structural scales become observer-side coefficients, steps, intervals, or parameters.
- Algebraic-Mapping Rules determine how admissibility or index relations become algebraic structures.
- Observer-Side Update Postulates determine the update form.
- Structural Constraints prevent overlay equations from becoming ontology.
- Every observer-side equation requires a traceability certificate.

Chapter 3

Memoryless Rarefaction Overlay

3.1 Declared Inputs

This chapter admits the first observer-side equation family under the Tier 2 rule registry. The overlay is memoryless, scalar, and non-adjacent. No index structure is invoked. No adjacency relation is invoked. No propagation parameter is introduced. The following declarations are invoked:

- SI1 — Structural Cadence Scale,
- MS1 — Cadence-to-Coefficient Map,
- D1 — Memoryless Proportional Update.

The following rule classes are not invoked:

- Algebraic-Mapping Rules: none invoked,
- Structural Constraints: none directly invoked.

No additional assumptions are introduced.

3.2 Overlay Declaration

Let (S(t)) denote a scalar observer-side representational quantity. The symbol (t) is an observer-side overlay parameter. It is not a Tier 1 time parameter. Under SI1, a structural cadence scale (f$_0$)$isadmittedasthestructuralscaleinput.UnderMS1,thecadencescalemaybemappedtoanobserver-sidecoefficient:[\kappa := af_0,]where(a)isadimensionlessoverlayparameter.Thecoefficient(\kappa)isrepresentationalo$
$[\frac{dS}{dt} = -\kappa S.]Thenegativesigndenotesdecreaseunderthedeclaredobserver-sideupdate.Nophysicalinterpretationis$

3.3 Solution

The declared observer-side equation [dS$\frac{}{dt=-\kappa S]hasthesolution[S(t)=S_0e^{-\kappa t},]where(S_0)istheinitialobserver-sidevalue.Substitutingt}$

3.4 Interpretive Boundary

The memoryless rarefaction overlay does not assert that any Tier 1 structure decays, disperses, relaxes, leaks, or undergoes probabilistic loss. It does not introduce a hazard function into the codex. It does not assert that structural cadence generates exponential behavior. The only claim made in this chapter is conditional:

> If a scalar observer-side quantity is declared to update memorylessly, and if a structural cadence scale is mapped to the proportional coefficient, then the admitted observer-side solution is exponential.

The equation family is therefore overlay-dependent.

3.5 Traceability Certificate

The admitted observer-side form is certified as follows: [$\boxed{\mathrm{SI1} + \mathrm{MS1} + \mathrm{MA}: \text{ none invoked} + \mathrm{D1} + \mathrm{SC}: \text{ none directl}}$] with [$\kappa := af_0 .]Thecorrespondingsolutionis[S(t) = S_0 e^{-\kappa t}.]Thiscertificateexhauststherulebasisoftheoverlay.$

Hierarchy Reminder

Chapter 3 Recall

- SI1 admits a structural cadence scale as input.
- MS1 maps cadence to the observer-side coefficient (κ).
- D1 declares memoryless proportional update.
- No adjacency algebra is invoked.
- No propagation parameter is introduced.
- No structural constraint is directly invoked.
- The exponential form is observer-side and overlay-dependent.
- No Tier 1 ontology is modified or extended.

Chapter 4

Symmetric Adjacency Overlay

4.1 Declared Inputs

This chapter admits the second observer-side equation under the rule registry of Chapter **??**.

The following declarations are invoked:

- Structural Inputs: SI1 (Structural Cadence Scale), SI2 (Structural Length Scale), SI3 (Structural Index Class).
- Algebraic-Mapping Rule MA1 (Discrete Adjacency Mapping).
- Scale-Mapping Rule MS2 (Scale-to-Step Identification).
- Observer-Side Update Postulate D2 (Symmetric Adjacency Update).
- Structural Constraints SC1–SC3 (in force).

No additional assumptions are introduced.

4.2 Discrete Update Rule

Let $\{P_n(t)\}$ denote a scalar representational quantity indexed by admissible lattice unit n.

Under MA1, the representational index admits adjacent indices $n-1$ and $n+1$. Under D2, adjacency is declared symmetric.

The discrete update rule is written:

$$P_n^{\text{new}} = \frac{1}{2}\left(P_{n-1} + P_{n+1}\right).$$

This rule specifies symmetric redistribution over adjacent representational indices. No motion, transfer, exchange, or physical interaction is asserted.

4.3 Increment Form

Define the update increment:

$$\Delta P_n = P_n^{\text{new}} - P_n.$$

Substituting the update rule:

$$\Delta P_n = \frac{1}{2}\left(P_{n-1} - 2P_n + P_{n+1}\right).$$

The quantity in parentheses is the centered second discrete difference admitted under MA2.

4.4 Continuum Identification

Under MS2, identify:

$$\Delta x = \alpha\lambda_0, \qquad \Delta t = \frac{\beta}{f_0},$$

where λ_0 and f_0 arise from the declared structural inputs, and α, β are dimensionless overlay parameters.

The update increment may be written:

$$\frac{\Delta P_n}{\Delta t} = \frac{1}{2}\frac{P_{n-1} - 2P_n + P_{n+1}}{\Delta t}.$$

Using the MS2 identification, this becomes:

$$\frac{\Delta P_n}{\Delta t} = \frac{\alpha^2\lambda_0^2}{2\Delta t}\frac{P_{n-1} - 2P_n + P_{n+1}}{\Delta x^2}.$$

The continuum limit is then taken as a representational approximation within the observer-side overlay:

$$\frac{\partial P}{\partial t} = D\frac{\partial^2 P}{\partial x^2},$$

where

$$D = \frac{\alpha^2\lambda_0^2}{2\Delta t} = \frac{\alpha^2}{2\beta}\lambda_0^2 f_0.$$

The continuum limit does not correspond to a Tier 1 structural operation. It is an approximation applied within the declared overlay.

4.5 Traceability

The diffusion form is admitted under the following traceability certificate:

$$\boxed{\mathrm{SI1}, \mathrm{SI2}, \mathrm{SI3} + \mathrm{MS2} + \mathrm{MA1}, \mathrm{MA2} + \mathrm{D2} + \mathrm{SC1}, \mathrm{SC2}, \mathrm{SC3} \Longrightarrow \frac{\partial P}{\partial t} = D\frac{\partial^2 P}{\partial x^2}}$$

The diffusion form arises solely from symmetric adjacency assumptions (MA1 + D2) together with scale-to-step identification (MS2).

It is an observer-side representational equation.

It does not assert physical diffusion, motion, transport, or exchange.

It does not introduce a Tier 1 dynamical law.

Hierarchy Reminder

Chapter 4 Recall

- Symmetric adjacency (MA1 + D2) produces second discrete differences.
- Scale-to-step identification (MS2) maps structural scales to observer-side steps.
- The continuum limit yields $\partial_t P = D\partial_x^2 P$ as an admitted overlay form.
- The diffusion coefficient D is a declared scale-mapping result.
- No Tier 1 geometric, dynamical, or transport claim is made.

Chapter 5

Persistent Update Overlay

5.1 Declared Inputs

This chapter admits the third observer-side equation under the rule registry of Chapter **??**.

The following declarations are invoked:

- Structural Inputs: SI1 (Structural Cadence Scale), SI2 (Structural Length Scale), SI3 (Structural Index Class).
- Algebraic-Mapping Rule MA1 (Discrete Adjacency Mapping).
- Scale-Mapping Rule MS2 (Scale-to-Step Identification).
- Scale-Mapping Rule MS5 (Persistence-Parameter Declaration).
- Observer-Side Update Postulate D2 (Symmetric Adjacency Update).
- Observer-Side Update Postulate D3 (Retained-Increment Update).
- Observer-Side Update Postulate D4 (Linear Retained-Update Damping).
- Structural Constraints SC1–SC3 (in force).

No additional assumptions are introduced.

5.2 Persistence Declaration

Let $P_n(t)$ denote a scalar representational quantity indexed by admissible lattice unit n.

Under MS5, a persistence parameter γ is declared.

Under D3, the observer-side update is permitted to depend on the previous update increment. This introduces second-order update structure in the time variable.

This declaration specifies update form only. It does not assert structural persistence, inertia, or memory at the Tier 1 level.

5.3 Discrete Form

Let Δt be the declared update interval.

The retained-increment structure is represented by the second discrete time difference:

$$\frac{P_n(t+\Delta t) - 2P_n(t) + P_n(t-\Delta t)}{\Delta t^2}.$$

Under MA1 and D2, the second discrete spatial difference is:

$$\frac{P_{n+1}(t) - 2P_n(t) + P_{n-1}(t)}{\Delta x^2}.$$

Under D4, a linear damping term is admitted:

$$2\gamma \frac{P_n(t) - P_n(t - \Delta t)}{\Delta t}.$$

Combining these declared forms yields:

$$\frac{P_n(t + \Delta t) - 2P_n(t) + P_n(t - \Delta t)}{\Delta t^2} = v^2 \frac{P_{n+1}(t) - 2P_n(t) + P_{n-1}(t)}{\Delta x^2} - 2\gamma \frac{P_n(t) - P_n(t - \Delta t)}{\Delta t}.$$

This is an admitted discrete observer-side update form.
No motion, transport, propagation, or interaction is asserted.

5.4 Continuum Identification

Under MS2, identify:

$$\Delta x = \alpha \lambda_0, \qquad \Delta t = \frac{\beta}{f_0},$$

where λ_0 and f_0 arise from the declared structural inputs, and α, β are dimensionless overlay parameters.

The continuum limit is then taken as a representational approximation within the observer-side overlay:

$$\frac{\partial^2 P}{\partial t^2} + 2\gamma \frac{\partial P}{\partial t} = v^2 \frac{\partial^2 P}{\partial x^2},$$

where

$$v = \frac{\alpha}{\beta} \lambda_0 f_0.$$

The continuum limit does not correspond to a Tier 1 structural operation.

5.5 Zero-Persistence Limit

Under the declared limit $\gamma \to 0$, the admitted form reduces to:

$$\frac{\partial^2 P}{\partial t^2} = v^2 \frac{\partial^2 P}{\partial x^2}.$$

This is the admitted observer-side wave form.
Second-order time structure arises solely from the retained-increment declaration (D3).

5.6 Traceability

The telegrapher and wave forms are admitted under the following traceability certificate:

$$\boxed{\mathrm{SI1}, \mathrm{SI2}, \mathrm{SI3} + \mathrm{MS2}, \mathrm{MS5} + \mathrm{MA1}, \mathrm{MA2} + \mathrm{D2}, \mathrm{D3}, \mathrm{D4} + \mathrm{SC1}, \mathrm{SC2}, \mathrm{SC3} \Longrightarrow \frac{\partial^2 P}{\partial t^2} + 2\gamma \frac{\partial P}{\partial t} = v^2 \frac{\partial^2 P}{\partial x^2}}$$

These equations arise solely from declared observer-side update rules.
They do not assert physical wave propagation, inertia, or dynamical law.

Hierarchy Reminder

Chapter 5 Recall

- Retained-increment update (D3) introduces second time differences.
- Symmetric adjacency (MA1 + D2) introduces second spatial differences.
- Linear damping (D4) yields telegrapher structure.
- $\gamma \to 0$ yields the wave form.
- No Tier 1 dynamical or ontological claim is made.

Chapter 6

Overlay Unification

6.1 Summary of Admitted Forms

Chapters **??**–5 have admitted the following observer-side equation families under explicitly declared update rules:

- Memoryless proportional update (D1)

  $$\Longrightarrow \frac{dS}{dt} = -\kappa S$$

  (Exponential form),
- Symmetric adjacency (MA1 + D2) with scale mapping (MS2)

  $$\Longrightarrow \frac{\partial P}{\partial t} = D\frac{\partial^2 P}{\partial x^2}$$

  (Diffusion form),
- Retained-increment update (D3) with symmetric adjacency (MA1 + D2) and scale mapping (MS2)

  $$\Longrightarrow \frac{\partial^2 P}{\partial t^2} = v^2\frac{\partial^2 P}{\partial x^2}$$

  (Wave form),
- Retained-increment update with damping (D3 + D4)

  $$\Longrightarrow \frac{\partial^2 P}{\partial t^2} + 2\gamma\frac{\partial P}{\partial t} = v^2\frac{\partial^2 P}{\partial x^2}$$

  (Telegrapher form).

Each form arises from a distinct declared observer-side update structure.
No equation is derived from structural grammar alone.

6.2 Common Structural Input

All admitted forms share the same structural inputs:

- SI1 (Structural Cadence Scale),
- SI2 (Structural Length Scale),

- SI3 (Structural Index Class).

These inputs supply dimensional coherence only.

They do not determine equation type.

Equation family type is determined entirely by declared update postulates.

6.3 Finite Update Classes

The admitted equation families correspond to a finite set of observer-side update structures:

- First-order memoryless update (D1),
- First-order symmetric adjacency update (D2),
- Second-order retained-increment update (D3),
- Retained-increment update with damping (D3 + D4).

All admitted observer-side equations in this volume arise from these update classes combined with declared scale mappings.

No additional equation class appears without introducing a new update postulate.

6.4 Non-Uniqueness of Overlay Realization

A fixed structural cadence scale supplies dimensional input only.

It does not select a unique observer-side equation.

Distinct equation families are compatible with the same structural input.

Therefore:

- Structural grammar does not uniquely determine observer-side equation form.
- Equation family selection is governed entirely by declared update structure.

This non-uniqueness reflects representational freedom under fixed structural input.

It does not reflect ambiguity in the structural grammar.

6.5 Separation Principle

Structural grammar remains predynamical and unaffected by observer-side overlay realizations.

Observer-side equations remain:

- Representational,
- Rule-dependent,
- Non-ontological.

No observer-side equation is promoted to Tier 1 structure.

6.6 Exhaustion Statement

Within the rule registry of Chapter **??**, all admitted observer-side equation families reduce to the finite set of update classes listed above.

No additional equation family may be introduced without declaring a new update postulate.

Thus:

Observer-side equation diversity is exhausted by declared update structure, not structural grammar.

Hierarchy Reminder

Chapter 6 Recall

- Exponential, diffusion, wave, and telegrapher forms arise from declared update postulates.
- All share the same structural inputs (SI1–SI3).
- Equation type depends on update structure, not structural grammar.
- A finite set of update classes exhausts observer-side equation families.
- Structural ontology and overlay representation remain separated.

Chapter 7

Overlay Scope Closure

This volume has introduced observer-side representational overlays operating on structural quantities defined in Papers I–V.

The following statements are fixed.

1. Structural Grammar Remains Unchanged

No structural primitive has been added. No admissibility condition has been modified. No torsion rule has been reinterpreted. No closure coefficient has been altered.

Structural grammar remains predynamical and time-independent.

2. Overlays Are Declared, Not Derived

All admitted observer-side equation families in this volume arise from explicitly declared:

- Structural Inputs (SI#),
- Scale-Mapping Rules (MS#),
- Algebraic-Mapping Rules (MA#),
- Observer-Side Update Postulates (D#),
- Structural Constraints (SC#).

No evolution equation has been presented as structurally inevitable.

Distinct update postulates applied to the same structural inputs admit distinct observer-side equation families.

Equation type depends on declared update structure, not on structural grammar.

3. No Ontological Promotion

Exponential, diffusion, telegrapher, and wave forms are admitted observer-side representations under declared overlays.

They are not structural necessities. They are not promoted to ontological status.

Structural admissibility does not imply a preferred dynamical realization.

4. Boundary with Dimensional Correlation

Dimensional correlation (Tier 3) operates on observer-side equations without modifying structural grammar.

Such procedures may:

- Eliminate explicit kilogram dependence,
- Preserve algebraic structure,
- Re-express dimensional relations.

They do not:

- Introduce structural primitives,
- Derive observer-side equation families,
- Establish Tier 1 admissibility conditions.

Dimensional reduction does not derive overlay equations. Overlay equations do not define structural grammar.

Final Statement

The overlay layer establishes admissible observer-side forms under declared update rules.

Structural ontology remains confined to Tier 1. Dimensional re-expression remains confined to Tier 3.

All observer-side dynamics in this volume are declared representations, not structural derivations.

The Bridge and the Boundary

Bridge to Dimensional Correlation

The Overlay Layer and the Dimensional Correlation Layer are formally independent but algebraically compatible.

Volume I admits observer-side equation families under explicitly declared Model Rules (M#), Dynamical Postulates (D#), and Structural Constraints (SC#). These equations arise solely from overlay declarations and do not follow from structural grammar alone.

Volume II performs lm-reduction on standard SI expressions. This procedure removes explicit kilogram dependence while preserving algebraic structure exactly. It introduces no structural primitives and no dynamical assumptions.

The interaction between the volumes is strictly limited:

- Overlay equations may be expressed in lm-reduced form.
- lm-reduction does not derive overlay equations.
- Overlay equations do not define structural primitives.
- Dimensional re-expression does not promote representational equations to ontology.

Tier separation is preserved across all volumes.

Document Status and Boundary

This document is a non-normative companion to the LMR predynamical program. It provides interpretive correspondence and structural framing derived from the codex established in Papers I–V.

It does not introduce new primitives, corridors, or admissibility conditions, and does not modify the codex grammar. All structural authority remains with the Arc 1 papers.

The present document is dependent on the codex and should be read only after the foundational sequence.

Readers are directed to the Arc 1 papers for governing definitions, or to the Concepts and Reference sections for structured navigation.

Companion Series Relation

Companion Volume I and Companion Volume II form a paired non-normative series within the LMR program.

Volume I introduces declared representational overlays operating on structural cadence scales. Volume II performs dimensional correlation of standard SI expressions under lm-reduction.

The two volumes are formally independent but algebraically compatible. Neither volume modifies structural grammar, and neither establishes ontological claims.

Together, they provide representational and dimensional context to the predynamical codex without altering its structure.